## CSE211: Compiler Design

 Oct. 4, 2023- Topic: Parsing overview 2

- Questions:
- What is a scanner?
- What are regular expressions? What are some use-cases for them?


## Announcements

- Piazza is up! Please enroll. It should be considered required!
- My office hours will be on Thursday 3-5 PM
- No hours this week though
- Occasional technical issues with recordings
- Will not be re-recording classes


## Announcements

- Homework 1 is planned for release on Monday by Midnight
- Please start thinking about partners
- Please self organize (use Piazza)
- You will have 2 weeks to do it
- Any remaining undergrads should get a permission code ASAP
- If anyone isn't on Canvas, please let me know


## Announcements

- Think about paper review
- You will need to approve a paper with me by Oct. 23
- First review is due Oct. 30
- You should probably not wait until these due dates because the midterm is also on Oct. 30.
- I give this time for you to organize, not as a guidance!
- You can discuss papers on piazza or ask me for suggestions

Review

## Scanner

- splits an input into tokens (e.g. parts of speech)



## Scanner

## My Old Computer Crashed

## Scanner

```
[(ARTICLE, "my") (ADJECTIVE, "old") (NOUN, "Computer") (VERB, "Crashed")]
```

Splits an input sentence it into lexemes

## Scanner

- Lets write tokens for arithmetic expression:

$$
(5+4) * 3
$$

## Scanner

$(5+4) * 3$
[[(LPAR, "(") (NUM, "5") (PLUS, "+") (NUM, "4") (RPAR, ")") (TIMES, "*") (NUM, "3")]

Splits an input sentence it into lexemes

## Defining tokens

- Literal - single character:
- PLUS = '+', TIMES = '*'
- Keyword - single string:
- IF = "if", INT = "int"
- Sets of words:
- NOUN = \{"Cat", "Dog", "Car"\}
- Numbers
- NUM = \{" 0 ", " 1 " ... $\}$


## Defining tokens

- Literal-single character:
-PLUS - ‘+', TIMES - ‘*'
- Keyword -single-string:
-IF = "if", INT = "int"
- Sets of words:
- NOUN - \{"Cat", "Dog", "Car" $\}$
- Numbers
- NUM - $\left\{{ }^{\prime \prime} 0 ", " 1 " \ldots\right\}$
- Regular expressions!


## Regular Expressions

- Lots of literature!
- Simplest grammar in the Chomsky language hierarchy
- abstract machine definition (finite automata)
- Many implementations (e.g. Python standard library)



## Regular Expressions

We will define RE's recursively:

Input:

- Regular Expression $R$
- String S


## Output:

- Does the Regular Expression $R$ match the string $S$


## Regular Expressions

We will define RE's recursively:

The base case: a character literal

- The RE for a character ' $x$ ' is given by ' $x$ '. It matches only the character ' $x$ '


## Regular Expressions

We will define RE's recursively:

Regular expressions are closed under concatenation:

- The concatenation of two REs $x$ and $y$ is given by $x y$ and matches the strings of RE $x$ concatenated with the strings of RE $y$


## Regular Expressions

We will define RE's recursively:

Regular expressions are closed under union:

- The union of two REs $x$ and $y$ is given by $x \mid y$ and matches the strings of RE $x$ OR the strings of RE $y$


## Regular Expressions

We will define RE's recursively:

Regular expressions are closed under Kleene star:

- The Kleene star of an RE x is given by $\mathrm{x}^{*}$ and matches the strings of RE $\times$ REPEATED 0 or more times


## Regular Expressions

Examples

## Regular Expressions

- Use ()'s to force precedence!
- Just like in math:
- $3+4$ * 5
- what is the precedence of concatenation, union, and star?
- "x \| yw"
- Is it "(x|y)w" or "x|(yw)"
- "xy*"
- is it (xy)* or $x\left(y^{*}\right)$


## Regular Expressions

- Use ()'s to force precedence!
- Just like in math:
- $3+4$ * 5
- what is the precedence of concatenation, union, and star?
- "x yw "
- Is it "(x|y)w" or "x|(yw)"
- "xy*"
- is it (xy)* or $x\left(y^{*}\right)$

How can we determine precedence?

## Regular Expressions

- Use ()'s to force precedence!
- Just like in math:
- $3+4$ * 5
- what is the precedence of concatenation, union, and star?
- Star > Concat > Union
- use () to avoid mistakes!


## Regular Expressions

Most RE implementations provide syntactic sugar:

- Ranges:
- [0-9]: any number between 0 and 9
- [a-z]: any lower case character
- [A-Z]: any upper case character
- Optional(?)
- Matches 0 or 1 instances:
- ab?c matches "abc" or "ac"
- can be implemented as: (abc | ac)


## Defining tokens using REs

- Literal - single character:
- PLUS = ‘+', TIMES = ‘*'

Why the backslash characters?

- Keyword - single string:
- IF = "if", INT = "int"
- Sets of words:
- NOUN = "(Cat)|(Dog)|(Car)"
- Numbers
- SINGLE_NUM = [0-9]
- how to do INT?
- how to do FLOAT?


## Defining tokens using REs

- Literal - single character:
- PLUS = ' $\backslash+$ ', TIMES = ‘${ }^{*}$ '
- Keyword - single string:
- IF = "if", INT = "int"
- Sets of words:
- NOUN = "(Cat)|(Dog)|(Car)"
- Numbers
- SINGLE_NUM = [0-9]
- INT = -?([1-9][0-9]*) | 0
- FLOAT =?


## Scanner

- Takes in a list of tokens and a string and tokenizes the input


## Scanner

## Input <br> "My Old Computer Crashed"

## Tokens

- ARTICLE = "The|A|My|Your"
- NOUN = "Dog|Car|Computer"
- VERB = "Ran|Crashed|Accelerated"
- ADJECTIVE = "Purple|Spotted|OId"
$[(A R T I C L E, \quad " m y ")$ (ADJECTIVE, "old") (NOUN, "Computer") (VERB, "Crashed")]

Tokens are defined with Regular expressions, which are used to split up the input stream into lexemes

## re.match

- A streaming API supported by most RE libraries
- Only has to match part the beginning part of the string, not the entire string


## re.match

- A streaming API supported by most RE libraries
- Only has to match part the beginning part of the string, not the entire string
- CLASS_TOKEN = \{"cse |211|cse211"\}
- What would get matched here?: "cse211"
-(CLASS_TOKEN, ?)


## Scanners should provide the longest possible match

- Important for operators, e.g. in C
- ++, +=,
how would we parse "x++;"
(ID, "x") (ADD, "+") (ADD, "+") (SEMI, ";")
(ID, "x") (INCREMENT, "++") (SEMI, ";")


## Subtle differences here

- RE definitions are not guaranteed to give you the longest possible match
- OP = "+|++", ID = " $[a-z] "$
- What will this return for " $\mathrm{x}++$ "
- Scanners will tokenize the string according to the token with the longest match
- PLUS = " + ", PP = " ++ ", ID = " $[a-z] "$
- What will this return for " $x++$ "
- What does this mean for you?
- If you are implementing a scanner?
- If you are writing tokens?


## Scanner Summary

- Tokens are defined using regular expressions
- A scanner uses tokens to split a string into lexemes
- Regular expressions are good for splitting up a program into numbers, variables, operators, and structure (e.g. parenthesis and braces)
- You will get more practice using them in the homework
- Chapter 2 in EAC goes into detail on regular expression parsing
- Finite automata etc.


## Define a full language using tokens?

-What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *

## Define a full language using tokens?

-What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *

- First let's define tokens:


## Define a full language using tokens?

-What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *

- First let's define tokens:
- NUM = [0-9]+
- PLUS = ${ }^{-1+}$ +
- TIMES = '\*'


## Define a full language using tokens?

-What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *

- First let's define tokens:
- NUM = [0-9]+
- PLUS = ' $\$ +
- TIMES = '\*'
- What should our language look like?


## Define a full language using tokens?

- What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *
- First let's define tokens:
- NUM = [0-9]+
- PLUS = ' $\$ +
- TIMES = '\*'
-What should our language look like?
- NUM


## Define a full language using tokens?

- What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *
- First let's define tokens:
- NUM = [0-9]+
- PLUS = ${ }^{-1+}$ +
- TIMES = '\*'
- What should our language look like?
- NUM
- NUM PLUS NUM


## Define a full language using tokens?

- What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *
- First let's define tokens:
- NUM = [0-9]+
- PLUS = ${ }^{\prime}+{ }^{\prime}$
- TIMES = ${ }^{\prime *}$
- What should our language look like?
- NUM
- NUM PLUS NUM


## Define a full language using tokens?

- What about a mathematical sentence (expression)?
limited to non-negative integers and just using + and *
- First let's define tokens:
- NUM = [0-9]+
- PLUS = ' $1+$ '
- TIMES = '\*'

Why not just use regular expressions?

What would the expression look like?

- What should our language look like?
- NUM
- NUM PLUS NUM


## Define a full language using tokens?

-Where are we going to run into issues?

## What about ()'s

- there is a formal proof available that regex CANNOT match ()'s: pumping lemma
- Informal argument:
- Try matching $\left({ }^{n}\right)^{n}$ using Kleene star
- Impossible!
- We are going to need a more powerful language description framework!


## What about ()'s

- there is a formal proof available that regex CANNOT match ()'s: pumping lemma
- Informal argument:
- Try matching $\left({ }^{n}\right)^{n}$ using Kleene star
- Impossible!

What other syntax like () are used in programming languages?

- We are going to need a more powerful language description framework!
(previously) $2^{\text {nd }}$ most upvoted post on stackoverflow


## Context Free Grammars

- Backus-Naur form (BNF)

- A syntax for representing context free grammars
- Naturally creates tree-like structures
- More powerful than regular expressions


## BNF Production Rules

- <production name> : <token list>
- Example: sentence: ARTICLE NOUN VERB
- <production name> : <token list> | <token list>
- Example:
sentence: ARTICLE ADJECTIVE NOUN VERB
| ARTICLE NOUN VERB
Convention: Tokens in all caps,
production rules in lower case


## BNF Production Rules

- Production rules can reference other production rules
sentence: non_adjective_sentence | adjective_sentence
non_adjective_sentence: ARTICLE NOUN VERB
adjective_sentence: ARTICLE ADJECTIVE NOUN VERB


## BNF Production Rules

sentence: ARTICLE ADJECTIVE* NOUN VERB

## BNF Production Rules

## sentence: ARTICLEADJECTIVE*NOUN VERB

We cannot do the star in production rules

## BNF Production Rules

- Production rules can be recursive
- Imagine a list of adjectives:
"The small brown energetic dog barked"
sentence: ARTICLE adjective_list NOUN VERB
adjective_list: ADJECTIVE adjective_list
| <empty>


## Let's go back to mathematical sentences (expressions)

- First lets define tokens:
- NUM = [0-9]+
- PLUS = ${ }^{-1+}$ +
- TIMES = '\*'


## Let's go back to mathematical sentences (expressions)

- First lets define tokens:
- NUM = [0-9]+
- PLUS = ' $\backslash+$ ’
- TIMES = '\*'
expression : NUM
| expression PLUS expression
| expression TIMES expression


## Let's go back to mathematical sentences (expressions)

- First lets define tokens:
- NUM = [0-9]+
- $\operatorname{PLUS}={ }^{\wedge}+{ }^{\prime}$
- TIMES = '\*'
expression : NUM
| expression PLUS expression
| expression TIMES expression


## Let's go back to mathematical sentences (expressions)

- First lets define tokens:
- NUM = [0-9]+
- $\operatorname{PLUS}={ }^{\wedge}+{ }^{\prime}$
- TIMES = '\*'
- LPAREN = `('
- RPAREN = ` ${ }^{\prime}$ '

What other syntax like () are used in programming
languages?
expression : NUM
| expression PLUS expression
| expression TIMES expression
| LPAREN expression RPAREN

How to determine if a string matches a CFG?

## Parse trees

- A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: 5
```

expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

## Parse trees

- A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: 5
```

expr: NUM
| expr PLUS expr expr
| expr TIMES expr
| LPAREN expr RPAREN

## Parse trees

- A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: 5
```

```
expr:NUM
    | expr PLUS expr expr root of the tree is
    | expr TIMES expr
        the entry production
    | LPAREN expr RPAREN
```


## Parse trees

- A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: 5
```

leafs are lexemes

```
```

    expr
    ```
    expr
    |
    |
<NUM,5>
```

<NUM,5>

```
expr: NUM
    | expr PLUS expr
    | expr TIMES expr
    | LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: 5*6

```
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: 5*6

```
expr: NUM
| expr PLUS expr expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: 5*6

```
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN


\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: 5*6

```
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN


\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: 5**6

```
\begin{tabular}{ll} 
expr : NUM & \\
& | expr PLUS expr \\
& expr \\
& | LPAREN \(\operatorname{expr}\) RPAREN
\end{tabular}

What happens in an error?

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
expr : NUM
\begin{tabular}{l} 
| expr PLUS expr \\
| expr TIMES expr \\
| LPAREN expr RPAREN
\end{tabular}

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.


Not possible!

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: (1+5)*6

```
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: (1+5)*6

```
expr: NUM
expr
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: (1+5)*6

```
\begin{tabular}{rl} 
expr \(:\) & NUM \\
& | expr PLUS expr \\
& | expr TIMES expr \\
& | LPAREN expr RPAREN
\end{tabular}


\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
expr : NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: (1+5)*6

```
\begin{tabular}{rl} 
expr \(:\) & NUM \\
& | expr PLUS expr \\
& | expr TIMES expr \\
& | LPAREN expr RPAREN
\end{tabular}


\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.


\section*{Parse trees}
- A string is accepted by a BNF form if and only if there exists a parse tree.
```

input: (1+5)*6

```
\begin{tabular}{rl} 
expr \(:\) & NUM \\
& | expr PLUS expr \\
& | expr TIMES expr \\
& | LPAREN expr RPAREN
\end{tabular}


\section*{Parse trees}
- Reverse question: given a parse tree: how do you create a string?


\section*{Ambiguous grammars}
"I saw a person on a hill with a telescope."

What does it mean??
https://www.quora.com/What-are-some-examples-of-ambiguous-sentences

\section*{Parse trees}
- Try making a parse tree from : \(1+5\) * 6
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN

\section*{Parse trees}
- Try making a parse tree from : \(1+5\) * 6


\section*{Parse trees}
- input: \(1+5\) * 6
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN


\section*{Parse trees}
- input: \(1+5\) * 6

expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN


\section*{Ambiguous grammars}
- What's the issue?


A string


Ambiguous grammars continue to the rest of compilation
- What's the issue?


A string


\section*{Meaning into structure}
- Structural meaning defined to be a post-order traversal

\section*{Meaning into structure}
- Structural meaning defined to be a post-order traversal
- Children return values to their parent
- Nodes are only evaluated once all their children have been evaluated
- Evaluated from left to right
- Also called "Natural Order"

\section*{Meaning into structure}
- Structural meaning defined to be a post-order traversal
- Children return values to their parent
- Nodes are only evaluated once all their children have been evaluated
- Evaluated from left to right
- Can also encode the order of operation

\section*{Ambiguous grammars}
-input: 1 + 5 * 6

expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN


\section*{Avoiding Ambiguity}
- How to avoid ambiguity related to precedence?
- Define precedence: ambiguity comes from conflicts. Explicitly define how to deal with conflicts, e.g. write* has higher precedence than +
- Some parser generators support this, e.g. Yacc

\section*{Avoiding Ambiguity}
- How to avoid ambiguity related to precedence?
- Second way: new production rules
- One rule for each level of precedence
- lowest precedence at the top
- highest precedence at the bottom
- Lets try with expressions and the following:
-+* ()

\section*{Avoiding Ambiguity}
- How to avoid ambiguity related to precedence?
- Second way: new production rules
- One rule for each level of precedence
- lowest precedence at the top
- highest precedence at the bottom
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
| term
\end{tabular} \\
\hline\(*\) & term & \begin{tabular}{l} 
: term TIMES term \\
| factor
\end{tabular} \\
\hline () factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
| NUM
\end{tabular} \\
\hline
\end{tabular}
- Lets try with expressions and the following:
-+*()

\section*{Now lets create a parse tree}
\[
\text { input: } 1+5 * 6
\]
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
| term
\end{tabular} \\
\hline * & term & \begin{tabular}{l} 
: term TIMES term \\
I factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}

\section*{Now lets create a parse tree}
input: 1+5*6
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
I term
\end{tabular} \\
\hline * & term & \begin{tabular}{l} 
: term TIMES term \\
I factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}

\section*{Now lets create a parse tree}
\[
\text { input: } 1+5 * 6
\]
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
I term
\end{tabular} \\
\hline * & term & \begin{tabular}{l} 
: term TIMES term \\
I factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}


\section*{Now lets create a parse tree}
input: 1+5*6
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l}
\(:\) expr PLUS expr \\
I term
\end{tabular} \\
\hline * & term & \begin{tabular}{l} 
: term TIMES term \\
I factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}


\section*{Now lets create a parse tree}
input: 1+5*6
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
| term
\end{tabular} \\
\hline * & term & \begin{tabular}{l}
\(:\) term TIMES term \\
I factor
\end{tabular} \\
\hline () & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
| NUM
\end{tabular} \\
\hline
\end{tabular}


\section*{Now lets create a parse tree}
input: 1+5*6
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
I term
\end{tabular} \\
\hline * & term & \begin{tabular}{l} 
: term TIMES term \\
I factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}


\section*{Now lets create a parse tree}
input: 1+5*6
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline+ & expr & \begin{tabular}{l} 
: expr PLUS expr \\
I term
\end{tabular} \\
\hline\(*\) & term & \begin{tabular}{l} 
: term TIMES term \\
| factor
\end{tabular} \\
\hline() & factor & \begin{tabular}{l} 
: LPAREN expr RPAREN \\
I NUM
\end{tabular} \\
\hline
\end{tabular}


\section*{Parsing REs}

Let's try it for regular expressions, \(\{\mid\). * ()\} (where . is concat)
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline & & \\
\hline & & \\
\hline & & \\
\hline & & \\
\hline
\end{tabular}

\section*{Parsing REs}

Let's try it for regular expressions, \(\{\mid\). * ()\} (where . is concat)
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline I & & \\
\hline . & & \\
\hline * & & \\
\hline () & & \\
\hline
\end{tabular}

\section*{Parsing REs}

Let's try it for regular expressions, \(\{\mid\). * ()\} (where . is concat)
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline I & union & \\
\hline . & concat & \\
\hline * & starred & \\
\hline () & unit & \\
\hline
\end{tabular}

\section*{Parsing REs}

Let's try it for regular expressions, \(\left\{\mid .{ }^{*}()\right\}\) (where . is concat)
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline I & union & \begin{tabular}{l} 
: union PIPE union \\
\(\mid\) concat
\end{tabular} \\
\hline concat & \begin{tabular}{l} 
: concat DOT concat \\
| starred
\end{tabular} \\
\hline * & starred & \begin{tabular}{l} 
: starred STAR \\
| unit
\end{tabular} \\
\hline () & unit & \begin{tabular}{l} 
: LPAREN union RPAREN \\
| CHAR
\end{tabular} \\
\hline
\end{tabular}

\section*{Parsing REs}

Let's try it for regular expressions, \(\left\{\mid .{ }^{*}()\right\}\)
input: a.b | c*
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline I & union & \begin{tabular}{l} 
: union PIPE union \\
| concat
\end{tabular} \\
\hline- & concat & \begin{tabular}{l} 
: concat DOT concat \\
| starred
\end{tabular} \\
\hline * & starred & \begin{tabular}{l} 
: starred STAR \\
| unit
\end{tabular} \\
\hline() & unit & \begin{tabular}{l} 
: LPAREN union RPAREN \\
\(\mid\) CHAR
\end{tabular} \\
\hline
\end{tabular}

\section*{Parsing REs}

Let's try it for regular expressions, \{| . * ()\}
input: a.b | c*
\begin{tabular}{|l|l|l|}
\hline Operator & Name & Productions \\
\hline I & union & \begin{tabular}{l} 
: union PIPE union \\
\(\mid\) concat
\end{tabular} \\
\hline. & concat & \begin{tabular}{l} 
: concat DOT concat \\
| starred
\end{tabular} \\
\hline * & starred & \begin{tabular}{l} 
: starred STAR \\
| unit
\end{tabular} \\
\hline () & unit & \begin{tabular}{l}
\(:\) LPAREN union RPAREN \\
| CHAR
\end{tabular} \\
\hline
\end{tabular}
```

