# CSE211: Compiler Design Nov. 20, 2023

• **Topic**: Loop structure and DSLs

- Discussion questions:
  - Lots of discussions throughout about loops and DSLs



#### Announcements

- Homework 3 is out
  - Due on Nov. 29 (1.5 weeks to do it)
  - Get a partner ASAP
- Start thinking about 2<sup>nd</sup> paper
- Final Project Getting close to the deadline to getting it approved
  - Approved in ~1 week (Nov. 27)!
  - Presentations must be ready by Dec. 6
  - Deadline is to get final project APPROVED, not start brainstorming
- One more homework assigned when HW 3 is due

#### Announcements

• HW 2 is graded, please let us know if there are issues ASAP

### Review identify safe loops to parallelize

### SMP Parallelism in For Loops

- Given a nest of *candidate* For loops, determine if we can we make the outer-most loop parallel?
  - Safely
  - efficiently
- Criteria: every iteration of the outer-most loop must be *independent* 
  - The loop can execute in any order, and produce the same result
- Such loops are called "DOALL" Loops. The can be flagged and handed off to another pass that can finely tune the parallelism (number of threads, chunking, etc)

- Criteria: every iteration of the outer-most loop must be *independent*
- How do we check this?
  - If the property doesn't hold then there exists 2 iterations, such that if they are re-ordered, it causes different outcomes for the loop.
  - Write-Write conflicts: two distinct iterations write different values to the same location
  - **Read-Write conflicts**: two distinct iterations where one iteration reads from the location written to by another iteration.

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

Calculate index based on i

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

Computation to store in the memory location

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

```
for (i = 0; i < size; i++) {
    a[index(i)] = loop(i);
}</pre>
```

for two distinct iterations:

 $i_x != i_y$ Check: index( $i_x$ ) != index( $i_y$ )

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

```
for (i = 0; i < size; i++) {
    a[index(i)] = loop(i);
}</pre>
```

for two distinct iterations:

 $i_x != i_y$ Check: index( $i_x$ ) != index( $i_y$ ) Because we start at 0 and increment by 1, we can use i to refer to loop iterations

- Criteria: every iteration of the outer-most loop must be *independent*
- the loop must produce the same result for any order of the iterations

*First example: write-write conflict* 

```
for (i = 0; i < size; i++) {
    a[index(i)] = loop(i);
}</pre>
```

for two distinct iterations:

 $i_x != i_y$ Check: index( $i_x$ ) != index( $i_y$ ) Why?
Because if
index(i<sub>x</sub>) == index(i<sub>y</sub>)
then:
a[index(i<sub>x</sub>)] will equal
either loop(i<sub>x</sub>) or loop(i<sub>y</sub>)
depending on the order

#### Automation?

• We have decent intuition about this, but if its going to be in a compiler, then it needs to be automatable

```
for (i = 0; i < 128; i++) {
    a[i]= a[i]*2;
}

two integers: i<sub>x</sub> != i<sub>y</sub>
    i<sub>x</sub> >= 0
    i<sub>x</sub> < 128
    i<sub>y</sub> >= 0
    i<sub>y</sub> < 128

write-write conflict write_index(i<sub>x</sub>) == write_index(i<sub>y</sub>)
write_index(i<sub>x</sub>) == read_index(i<sub>y</sub>)
```

Ask if these constraints are satisfiable (if so, it is not safe to parallelize)

### SMT Solver

- Satisfiability Modulo Theories (SMT)
  - Generalized SAT solver
- Solves many types of constraints over many domains
  - Integers
  - Reals
  - Bitvectors
  - Sets
- Complexity bounds are high (and often undecidable). In practice, they work pretty well

#### Discussion: Data races

#### Are data races ever okay?

#### Are data races ever okay?

• Consider this program:

What can go wrong if we run the loop in parallel?

December 28, 2011 Volume 9, issue 12

🔁 PDF

#### You Don't Know Jack about Shared Variables or Memory Models

Data races are evil.

Hans-J. Boehm, HP Laboratories, Sarita V. Adve, University of Illinois at Urbana-Champaign

#### The final count

can also be too high. Consider a case in which the count is bigger than a machine word. To avoid dealing with binary numbers, assume we have a decimal machine in which each word holds three digits, and the counter x can hold six digits. The compiler translates x++ to something like

#### Now assume that x

is 999 (i.e.,  $x_hi = 0$ , and  $x_lo = 999$ ), and two threads, a blue and a red one, each increment x as follows (remember that each thread has its own copy of the machine registers tmp\_hi and tmp\_lo):

tmp_hi = x_hi;	
<pre>tmp_lo = x_lo;</pre>	
<pre>(tmp_hi, tmp_lo)++;</pre>	$//tmp_hi = 1, tmp_lo = 0$
<pre>x_hi = tmp_hi;</pre>	$//x_hi = 1, x_lo = 999, x = 1999$
x++;	<pre>//red runs all steps</pre>
	$//x_{hi} = 2, x_{lo} = 0, x = 2000$
<pre>x_lo = tmp_lo;</pre>	$//x_{hi} = 2, x_{lo} = 0$

#### Horrible data races in the real world

Therac 25: a radiation therapy machine

- Between 1987 and 1989 a software bug caused 6 cases where radiation was massively overdosed
- Patients were seriously injured and even died.
- Bug was root caused to be a data race.
- https://en.wikipedia.org/wiki/Therac-25

#### Horrible data races in the real world

2003 NE power blackout

- second largest power outage in history: 55 million people were effected
- NYC was without power for 2 days, estimated 100 deaths
- Root cause was a data race
- https://en.wikipedia.org/wiki/Northeast\_blackout\_of\_2003

# But checking for data conflicts is hard...

- Tools are here to help (Professor Flanagan is famous in this area)
- My previous group:
  - "Dynamic Race Detection for C++11" Lidbury and Donaldson
  - Scalable (complete) race detection
    - Firefox has ~40 data races
    - Chromium has ~6 data races

#### How to efficiently parallelize loops

# Shifting our focus back to a single core

- We need to consider single threaded performance
- Good single threaded performance can enable better parallel performance
  - Memory locality is key to good parallel performance.



#### Shifting our focus back to a single core

• Why?

Scalability! But at what COST? Derek G. Murray Unaffiliated<sup>†</sup> Michael Isard Unaffiliated\* Frank McSherry Unaffiliated 1000 50 econds system B system B We offer a new metric for big data platforms, COST, dn-paads 100 300 Abstract or the Configuration that Outperforms a Single Thread. 10 cores The COST of a given platform for a given problem is the Figure 1: Scaling and performance measurements 100 300 hardware configuration required before the platform outfor a data-parallel algorithm, before (system A) and after (system B) a simple performance optimization. performs a competent single-threaded implementation. The unoptimized implementation "scales" far better, COST weighs a system's scalability against the overheads introduced by the system, and indicates the actual despite (or rather, because of) its poor performance. performance gains of the system, without rewarding systems that bring substantial but parallelizable overheads. While this may appear to be a contrived example, we will We survey measurements of data-parallel systems reargue that many published big data systems more closely cently reported in SOSP and OSDI, and find that many systems have either a surprisingly large COST, often states or simply underperform one thread

# Shifting our focus back to a single core

#### • Why?

#### **1** Introduction

"You can have a second computer once you've shown you know how to use the first one."

–Paul Barham

1 1 1 .		•••	1 2007 05
scalable system	cores	twitter	uk-2007-05
GraphChi [12]	2	3160s	6972s
Stratosphere [8]	16	2250s	-
X-Stream [21]	16	1488s	-
Spark [10]	128	857s	1759s
Giraph [10]	128	596s	1235s
GraphLab [10]	128	249s	833s
GraphX [10]	128	419s	462s
Single thread (SSD)	1	300s	651s
Single thread (RAM)	1	275s	-

Table 2: Reported elapsed times for 20 PageRank iterations, compared with measured times for singlethreaded implementations from SSD and from RAM. GraphChi and X-Stream report times for 5 Page-Rank iterations, which we multiplied by four.

- Locality is key for good (parallel) performance:
- What kind of locality are we talking about?

- Locality is key for good parallel performance:
- Two types of locality:
  - Temporal locality
  - Spatial locality



- Locality is key for good parallel performance:
- Two types of locality:
  Temporal locality
  Spatial locality
  r1 = a[2];
  r2 = a[3];

how far apart can memory locations be?

• Locality is key for good (parallel) performance:

good data locality: cores will spend most of their time accessing private caches



• Locality is key for good (parallel) performance:

Bad data locality: cores will pressure and thrash shared memory resources





Row major



Row major



Row major


Column major? Fortran Matlab R



Column major? Fortran Matlab R



say x == y == 0

x1 = a[x,y]; x2 = a[x, y+1];

good pattern for row major bad pattern for column major

unrolled row major: still has locality

x1 = a[x,y]; x2 = a[x, y+1];

good pattern for row major bad pattern for column major

x1 = a[x,y]; x2 = a[x, y+1];

good pattern for row major bad pattern for column major unrolled column major: Bad locality



say x == y == 0

x1 = a[x,y]; x2 = a[x+1, y];

good pattern for column major bad pattern for row major

row major unrolled: bad spatial locality

unrolled

column

major:

x1 = a[x, y]; $x^{2} = a[x+1, y];$ 

good pattern for column major bad pattern for row major



#### How much does this matter?

which will be faster? by how much?

#### Demo

# How to reorder loop nestings?

- For a loop when can we reorder loop nestings?
  - If loop iterations are independent
  - If loop bounds are independent

# How to reorder loop nestings?

- For a loop when can we reorder loop nestings?
  - If loop iterations are independent
  - If loop bounds are independent
- If the loop bounds are dependent...

bad nesting order for row-major!

bad nesting order for row-major!

but iteration variables are dependent

bad nesting order for row-major!

but iteration variables are dependent

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

х = у

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

У

System with N variables can be viewed as an N dimensional polyhedron



# Fourier-Motzkin elimination:

- Given a system of inequalities with N variables, reduce it to a system with N 1 variables.
- A system of inequalities describes an N-dimensional polyhedron. Produce a system of equations that projects the polyhedron onto an N-1 dimensional space

x = y

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

System with N variables can be viewed as an N dimensional polyhedron



# Fourier-Motzkin elimination:

• To eliminate variable  $x_i$ :

For every pair of lower bound  $L_i$  and upper bound  $U_i$  on  $x_i$ , create:

 $L_i \leq x_i \leq U_i$  Then simply remove  $x_i$ :

 $L_i \leq U_i$ 

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

All pairs of upper/lower bounds on y:

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

0 <= y <= 5

0 <= y <= x

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
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}</pre>
```

All pairs of upper/lower bounds on y:

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

```
0 <= y <= 5
0 <= y <= x
```

Then eliminate y:

0 <= 5 0 <= x

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

All pairs of upper/lower bounds on y:

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

0 <= y <= 5 0 <= y <= x

Then eliminate y:

0 <= 5 0 <= x

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

All pairs of upper/lower bounds on y:

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

0 <= y <= 5 0 <= y <= x

Then eliminate y:

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

All pairs of upper/lower bounds on y:

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

 $0 \le y \le 5$  $0 \le y \le x$ 

x >= 0 x <= 7

loop constraints without y:

Then eliminate y:

0 <= x

x = y

loop constraints
y >= 0
y <= 5
x >= y
x <= 7</pre>

System with N variables can be viewed as an N dimensional polyhedron



### Reording Loop bounds:

- Given a new order:  $[x_0, x_1, x_2, ..., x_n]$
- For each variable  $x_i$ : perform Fourier-Motzkin elimination to eliminate any variables that come after  $x_i$  in the new order.
- Instantiate loop conditions for x<sub>i</sub>, potentially using max/min operators

loop constraints y >= 0 y <= 5 x >= y

```
for (y = 0; y <= 5; y++) {
  for (x = y; x <= 7; x++) {
     a[x,y] = b[x,y] + c[x,y];
  }
}</pre>
```

new order: [x,y]

for x: eliminate y using FM elimination:

loop constraints  $y \ge 0$   $y \le 5$  $x \ge y$ 

new order: [x,y]

for x: eliminate y using FM elimination:

x loop constraints without y:

x >= 0 x <= 7

loop constraints  $y \ge 0$   $y \le 5$  $x \ge y$ 

new order: [x,y]

for x: eliminate y using FM elimination:

x loop constraints without y:

x >= 0 x <= 7

y loop constraints: y >= 0

loop constraints

y >= 0

х >= у

new order: [x,y]

for x: eliminate y using FM elimination:

x loop constraints without y:

x >= 0 x <= 7

y loop constraints:

y >= 0 y <= 5

<mark>у <= х</mark>

y <= 5 x >= y

y >= 0

loop constraints

new order: [x,y]

for x: eliminate y using FM elimination:

x loop constraints without y:

x >= 0 x <= 7

y loop constraints: y >= 0 y <= min(x,5)</pre>

loop constraints

- y >= 0
- y <= 5
- х >= у
- x <= 7

У

x loop constraints without y:

x >= 0 x <= 7

y loop constraints: y >= 0 y <= min(x,5)</pre>



Х

# Reordering loop bounds

- only works if loop increments by 1; assumes a closed polyhedron
- best performance when array indexes are simple:
  - **e.g.:** a [x, y]
  - harder with, e.g.: a [x\*5+127, y+x\*37]
  - There exists schemes to automatically detect locality. Reach chapter 10 of the Dragon book
- compiler implementation allows exploration and auto-tuning
In some cases, there might not be a good nesting order for all accesses:

 $A = B + C^T$ 

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В







cold miss for all of them

Α

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В



Hit on A and B. Miss on C

Α





С

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В



Α





С

Hit on A and B. Miss on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

A

$$A = B + C^T$$







 Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$







• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

 $A = B + C^T$ 







cold miss for all of them

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

A

 $A = B + C^T$ 



С



Miss on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$



A





Miss on A,B, hit on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$



Α





Hit on all!

```
for (int x = 0; x < SIZE; x++) {
    for (int y = 0; y < SIZE; y++) {
        a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
     }
}</pre>
```

```
for (int xx = 0; xx < SIZE; xx += B) {
  for (int yy = 0; yy < SIZE; yy += B) {
    for (int x = xx; x < xx+B; x++) {
      for (int y = yy; y < yy+B; y++) {
         a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
      }
    }
}</pre>
```

```
for (int xx = 0; xx < SIZE; xx += B) {
  for (int yy = 0; yy < SIZE; yy += B) {
    for (int x = xx; x < xx+B; x++) {
      for (int y = yy; y < yy+B; y++) {
        a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
      }
    }
}</pre>
```

```
for (int xx = 0; xx < SIZE; xx += B) {
  for (int yy = 0; yy < SIZE; yy += B) {
    for (int x = xx; x < xx+B; x++) {
      for (int y = yy; y < yy+B; y++) {
        a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
      }
    }
}</pre>
```

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for (int xx = 0; xx < SIZE; xx += B) {
  for (int yy = 0; yy < SIZE; yy += B) {
    for (int x = xx; x < xx+B; x++) {
      for (int y = yy; y < yy+B; y++) {
         a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
      }
    }
}</pre>
```

### Recap what we've covered with loops

- Are the loop iterations independent?
  - The property holding all of these optimizations together
- mainstream compilers don't do much to help us out here
  why not?
- But DSLs can!

### Discussion

### **Discussion questions**:

What is a DSL? What are the benefits and drawbacks of a DSL? What DSLs have you used?

### What is a DSL

- Objects in an object oriented language?
  - operator overloading (C++ vs. Java)
- Libraries?
  - Numpy
- Does it need syntax?
  - Pytorch/Tensorflow

### What is a DSL

- Not designed for general computation, instead designed for a domain
- How wide or narrow can this be?
  - Numpy vs TensorFlow
  - Pros and cons of this design?
- Domain specific optimizations
  - Optimizations do not have to work well in all cases

• Ease of expressiveness

sed `s/Utah/California' address.txt

#### gnuplot

```
set title "Parallel timing experiments"
set xlabel "Threads"
set ylabel "Speedup"
plot "data.dat" with lines
```

Other examples?

These require their own front end. What about Matplotlib?

• Ease of expressiveness

make it harder to write bugs!







(a) Correct implementation. (b) With geometry bug.

• Ease of optimizations

Examples?

### From homework 3:

What does this assume? Optional in C++ *Non-optional in Tensorflow* 

- reduction loops:
  - Entire computation is dependent
  - Typically short bodies (addition, multiplication, max, min)



addition: 21 max: 6

min: 1

Typically faster than implementations in general languages.

• Easier to reason about

gnuplot example again

```
set title "Parallel timing experiments"
set xlabel "Threads"
set ylabel "Speedup"
plot "data.dat" with lines
```

Typically much fewer lines of code than implementations in general languages. tensorflow

tf.matmul(a, b)

What does an optimized matrix multiplication look like?

https://github.com/flame/blis/tree/master/kernels

- Easier to maintain
- Optimizations and transforms are less general (more targeted).
- Less syntax (sometimes no syntax).
- Fewer corner cases.

- Recipe for a DSL talk:
  - Introduce your domain
  - Show scary looking optimized code
  - Show clean DLS code
  - Show performance improvement
  - Have a correctness argument

# The rest of the lecture

- A discussion and overview of Halide:
  - Huge influence on modern DSL design
  - Great tooling
  - Great paper
- Originally: A DSL for image pipelining:





Brighten example

### Motivation:





pretty straight forward computation for brightening

(1 pass over all pixels)

This computation is known as the "Local Laplacian Filter". Requires visiting all pixels 99 times





We want to be able to do this fast and efficiently!

Main results in from Halide show a 1.7x speedup with 1/5 the LoC over hand optimized versions at Adobe

# Decoupling computation from optimization

- We love Halide not only because it can make pretty pictures very fast
- We love it because it changed the level of abstraction for thinking about computation and optimization
- (Halide has been applied in many other domains now, turns out everything is just linear algebra)

# Example

### • in C++

Which one would you write?

# Optimizations are a black box

- What are the options?
  - -00, -01, -02, -03
  - Is that all of them?
  - What do they actually do?

https://stackoverflow.com/questions/15548023/clang-optimization-levels

# Optimizations are a black box

- What are the options?
  - -00, -01, -02, -03
  - Is that all of them?
  - What do they actually do?
- **Answer**: they do their best for a wide range of programs. The common case is that you should not have to think too hard about them.
- *In practice*, to write high-performing code, you are juggling computation and optimization in your mind!

# Halides approach

- Decouple
  - what to compute (the program)
  - with how to compute (the optimizations, also called the schedule)

# Halides approach

- Decouple
  - what to compute (the program)
  - with how to compute (the optimizations, also called the schedule)

program add(x,y) = b(x,y) + c(x,y)schedule

add.order(x,y)

Halide (high-level)

# Halides approach

- Decouple
  - what to compute (the program)
  - with how to compute (the optimizations, also called the schedule)

program
add(x,y) = b(x,y) + c(x,y)
schedule
add.order(x,y)

Halide (high-level)

Pros and Cons?

### Halide optimizations

- Now all of a sudden, the programmer has to worry about how to optimize the program. Previously the compiler compiler made those decisions and we just "helped".
- What can we do here?
### Halide optimizations

- Auto-tuning
  - automatically select a schedule
  - compile and run/time the program.
  - Keep track of the schedule that performs the best
- Why don't all compilers do this?

### Halide optimizations

- Auto-tuning
  - automatically select a schedule
  - compile and run/time the program.
  - Keep track of the schedule that performs the best
- Why don't all compilers do this?
- Image processing is especially well-suited for this:
  - Images in different contexts might have similar sizes (e.g. per phone, on twitter, on facebook)

### Halide programs

- Halide programs:
  - built into C++, contained within a header

#include "Halide.h"

Halide::Func gradient; // a pure function declaration

Halide::Var x, y; // variables to use in the definition of the function (types?)

gradient (x, y) = x + y; // the function takes two variables (coordinates in the image) and adds them



increasing







increasing

what are some properties of this computation?



after applying the gradient function



increasing

what are some properties of this computation?Data races?Loop indices and increments?The order to compute each pixel?

#### Executing the function

Halide::Buffer<int32\_t> output = gradient.realize({3, 3});

Not compiled until this point Needs values for x and y



### Example: brightening





Brighten example

```
Halide::Buffer<uint8_t> input = load image("parrot.png");
```

```
Halide::Func brighter;
```

```
Halide::Expr value = input(x, y, c);
```

```
value = Halide::cast<float>(value);
```

```
value = value * 1.5f;
```

```
value = Halide::min(value, 255.0f);
```

```
value = Halide::cast<uint8_t>(value);
```

```
brighter(x, y, c) = value;
```

```
Halide::Buffer<uint8_t> input = load_image("parrot.png");
```

```
Halide::Func brighter;
```

```
Halide::Expr value = input(x, y, c);
```

```
value = Halide::cast<float>(value);
```

```
value = value * 1.5f;
```

```
value = Halide::min(value, 255.0f);
```

```
value = Halide::cast<uint8_t>(value);
```

```
brighter(x, y, c) = value;
```

brighter(x, y, c) = Halide::cast<uint8\_t>(min(input(x, y, c) \* 1.5f, 255));

#### Schedules

```
Halide::Func gradient;
Halide::Var x, y;
gradient(x, y) = x + y;
Halide::Buffer<int32_t> output =
    gradient.realize({3, 3});
```



increasing

```
Halide::Func gradient;
Halide::Var x, y;
gradient(x, y) = x + y;
Halide::Buffer<int32_t> output =
    gradient.realize({4, 4});
```

```
for (int y = 0; y < 4; y++) {
    for (int x = 0; x < 4; x++) {
        output[y,x] = x + y;
    }
}</pre>
```

```
Halide::Func gradient;
Halide::Var x, y;
gradient(x, y) = x + y;
Halide::Buffer<int32_t> output =
      gradient.realize({4, 4});
```



```
for (int y = 0; y < 4; y++) {
    for (int x = 0; x < 4; x++) {
        output[y,x] = x + y;
     }
}</pre>
```

Halide::Func gradient; Halide::Var x, y; gradient(x, y) = x + y; Halide::Buffer<int32\_t> output = gradient.realize({4, 4});

#### Schedule

gradient.reorder(y, x);

```
for (int x = 0; x < 4; x++) {
    for (int y = 0; y < 4; y++) {
        output[y,x] = x + y;
    }
}</pre>
```

#### Schedule

gradient.reorder(y, x);



```
for (int x = 0; x < 4; x++) {
    for (int y = 0; y < 4; y++) {
        output[y,x] = x + y;
    }
}</pre>
```

Halide::Func gradient; Halide::Var x, y; gradient(x, y) = x + y; Halide::Buffer<int32\_t> output = gradient.realize({4, 4});

#### Schedule

Var x\_outer, x\_inner; gradient.split(x, x\_outer, x\_inner, 2);

```
for (int y = 0; y < 4; y++) {
    for (int x_outer = 0; x_outer < 2; x_outer++) {
        for (int x_inner = 0; x_inner < 2; x_inner++) {
            x = x_outer*2 + x_inner;
            output[y,x] = x + y;
        }
    }
}</pre>
```

# Tiling

 In some cases, there might not be a good nesting order for all accesses:

 $A = B + C^T$ 

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В







cold miss for all of them

Α

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В



Hit on A and B. Miss on C

Α





С

 In some cases, there might not be a good nesting order for all accesses:

$$A = B + C^T$$

В



Α





С

Hit on A and B. Miss on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

A

$$A = B + C^T$$







 Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$







• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

 $A = B + C^T$ 







cold miss for all of them

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

A

 $A = B + C^T$ 



С



Miss on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$



Α





Miss on A,B, hit on C

• Blocking operates on smaller chunks to exploit locality in column increment accesses. Example 2x2

$$A = B + C^T$$



Α





Hit on all!

```
for (int x = 0; x < SIZE; x++) {
    for (int y = 0; y < SIZE; y++) {
        a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
     }
}</pre>
```

transforms into:

```
for (int xx = 0; xx < SIZE; xx += B) {
  for (int yy = 0; yy < SIZE; yy += B) {
    for (int x = xx; x < xx+B; x++) {
      for (int y = yy; y < yy+B; y++) {
         a[x*SIZE + y] = b[x*SIZE + y] + c[y*SIZE + x];
      }
    }
}</pre>
```

#### Schedule

Var x\_outer, x\_inner, y\_outer, y\_inner; gradient.split(x, x\_outer, x\_inner, 4); gradient.split(y, y\_outer, y\_inner, 4); gradient.reorder(x\_inner, y\_inner, x\_outer, y\_outer);

```
for (int y = 0; y < 16; y++) {
    for (int x = 0; x < 16; x++) {
        output[y,x] = x + y;
    }
}</pre>
```



#### Schedule

Var x\_outer, x\_inner, y\_outer, y\_inner; gradient.split(x, x\_outer, x\_inner, 4); gradient.split(y, y\_outer, y\_inner, 4); gradient.reorder(x\_inner, y\_inner, x\_outer, y\_outer);

```
for (int y = 0; y < 4; y++) {
    for (int x = 0; x < 4; x++) {
        output[y,x] = x + y;
     }
}</pre>
```

#### Parallelism?

• Next lecture

#### Next class

- Continuing on DSL parallelism
- See you on Thursday
- Get a partner for homework 3!