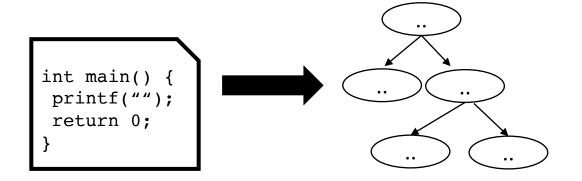
CSE211: Compiler Design

Sept. 27, 2022

• **Topic**: Parsing



- Questions:
 - What is parsing?
 - Have you used Regular Expressions before?
 - How do you parse Regular Expressions? What about Context-free Grammars?

Enrollment

- Got most people in!
 - but class is closed now
- Class is up to 30 with an original cap of 19. This changes things!
 - We have some undergrad graders to help
 - Office hours will likely be fairly busy. There will not be additional tutoring available.
 - Assignments will done in pair programming.
 - This is not just to make assignments easier or grading easier, but for you to tutor each other!
 - Please pick a new partner for each assignment
 - Any programming work towards the assignment must be done together!! (zoom with shared screen is fine).

Attendance

- Please mark yourself present on the dates you attend
 - I'll send out a google sheet later today
- Please don't cheat.
- If you have questions, just ask!

Piazza

• I have a piazza set up, I will email out the link later

New students

• If you were not here the first day of class, please watch the recording.

Office hours

Moved to Friday this week

Review

• What is a compiler?

Review

• What is a compiler?

• Besides translations, what else can a compiler do?

Review

• What is a compiler?

- Besides translations, what else can a compiler do?
 - analysis
 - optimizations

What can happen when the Input isn't valid?

```
int my var = 5;
my var = my car + 5;
int foo() {
  int *x = malloc(100*sizeof(int))
  return x[100];
```

What about this one?

Can the compiler make your code go faster?

```
int my_var = 0;
for (int i = 0; i < 128; i++) {
    my_var++;
}</pre>
```

Try running this on https://godbolt.org/ change the optimization level to -O3 and see what happens!

What is the compiler allowed to do?

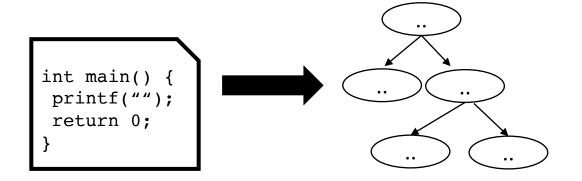
```
void add_arrays(int *a, int *b)
for (int i = 0; i < 128; i++) {
    a[i] += b[i];
}</pre>
```

Try running this on https://godbolt.org/ change the optimization level to -O3 and see what happens! Look for instructions like paddd. what does it do?

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• **Topic**: Parsing

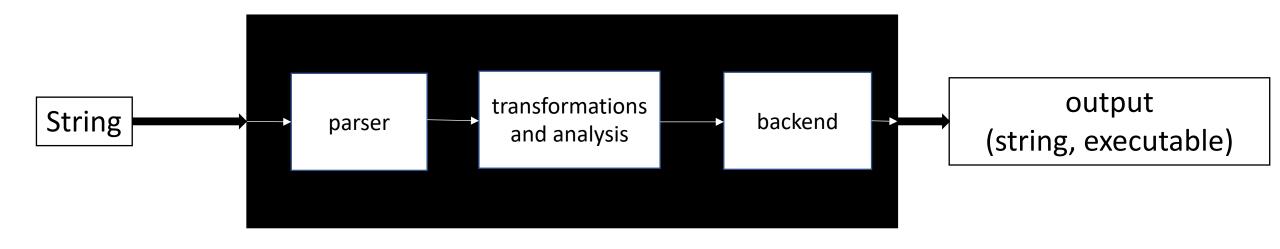


- Questions:
 - What is parsing?
 - Have you used Regular Expressions before?
 - How do you parse Regular Expressions? What about Context-free Grammars?

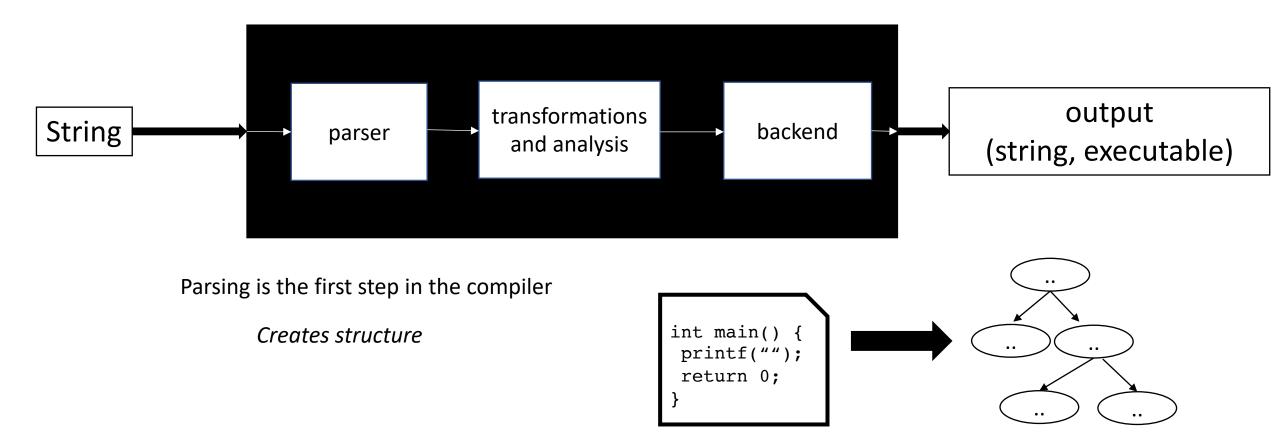
Compiler architecture overview



Compiler architecture overview



Compiler architecture overview

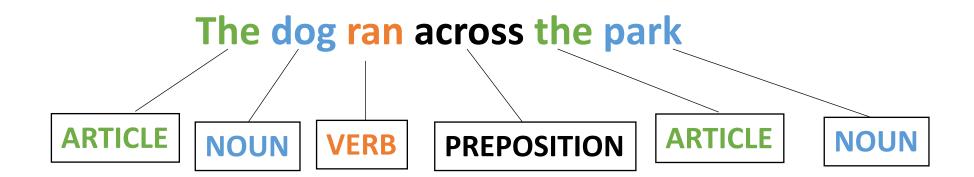


How do we parse a sentence in English?

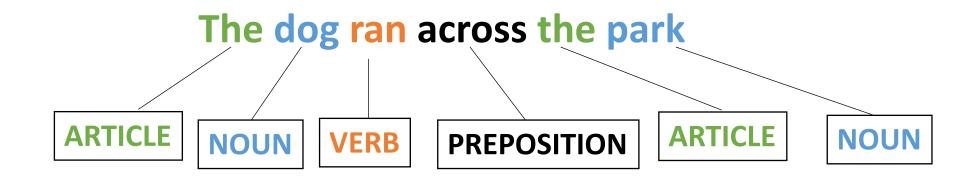
How do we parse a sentence in English?

The dog ran across the park

How do we parse a sentence in English?



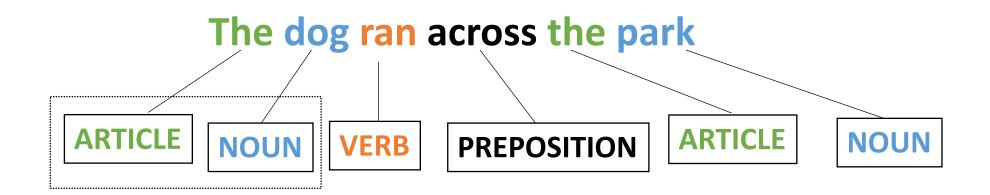
How do we parse a sentence in English?



Grammar and Syntax

What about semantics?

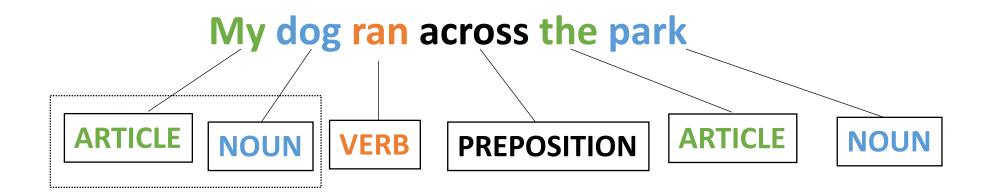
How do we parse a sentence in English?



Grammar and Syntax

What about semantics?

How do we parse a sentence in English?



Grammar and Syntax

What about semantics?

New Question

Can we define a simple language using these building blocks?

- ARTICLE
- NOUN
- VERB
- ADJECTIVE

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

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ARTICLE NOUN VERB

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ARTICLE ADJECTIVE NOUN

VERB

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

Question mark means optional

ARTICLE ADJECTIVE? NOUN

VERB

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

ARTICLE ADJECTIVE? NOUN VERB

My Old Computer Crashed

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

ARTICLE ADJECTIVE? NOUN VERB

The Purple Dog Crashed

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

Syntactically correct, logically correct?

ARTICLE ADJECTIVE? NOUN VERB

The Purple Dog Crashed

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

What other sentences can you construct?

ARTICLE ADJECTIVE? NOUN

VERB

- ARTICLE = {The, A, My, Your}
- NOUN = {Dog, Car, Computer}
- VERB = {Ran, Crashed, Accelerated}
- ADJECTIVE = {Purple, Spotted, Old}

What other languages can you specify?

ARTICLE ADJECTIVE* NOUN VERB

Goals in this module

- **Understand** the architecture of a modern parser (*tokenizing and parsing*)
- **Understand** the language of tokens (*regular expressions*) and parsers (*context-free grammars*)
- How to design CFG production rules so avoid ambiguity and encode precedence and associativity.
- Utilize a classic parser generator (Lex and Yacc) for a simple language

Goals in this module

• We will **NOT** discuss parsing algorithms for CFGs. If you are interested, you can do this for a paper assignment.

• This module should provide you with the background to implement parsers, which are **USEFUL** in many different projects.

 These topics are typically covered in more depth in an undergrad course.

High-level parser

Parser

High-level parser

A parser needs to know about the language:

• What forms can these take?

Parser

A parser needs to know about the language:

- 1800 page C++ specification,
 - English language
- Formal specification, mathematical
 - Mostly used in academics
 - X86, ARM, Functional languages

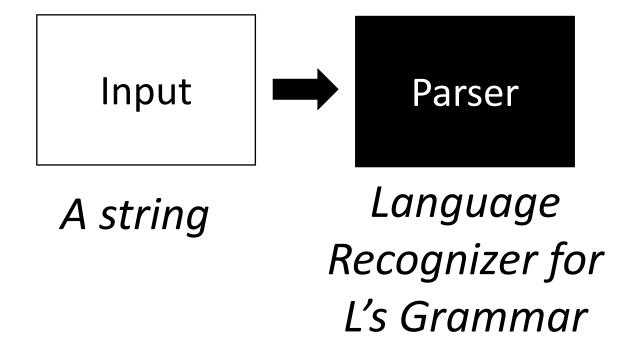
Parser

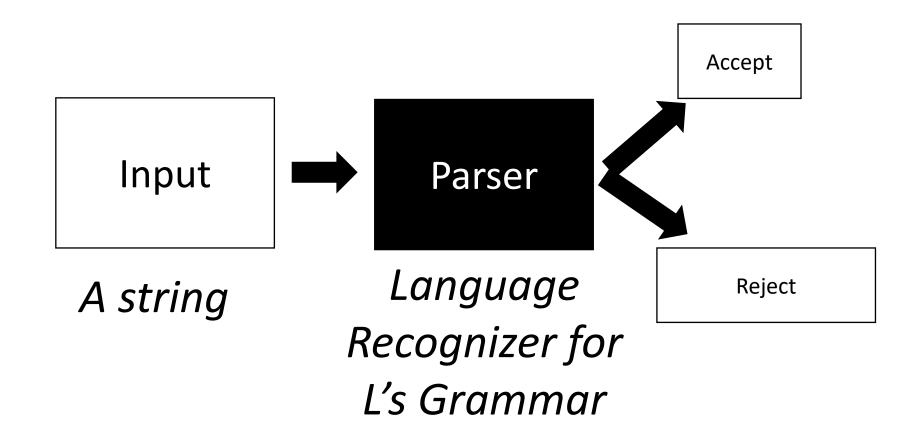
A parser needs to know about the language:

- 1800 page C++ specification,
 - English language
- Formal specification, mathematical
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 - X86, ARM, Functional languages

Parser

Parser needs only a small part of the specification!
The **Grammar!**

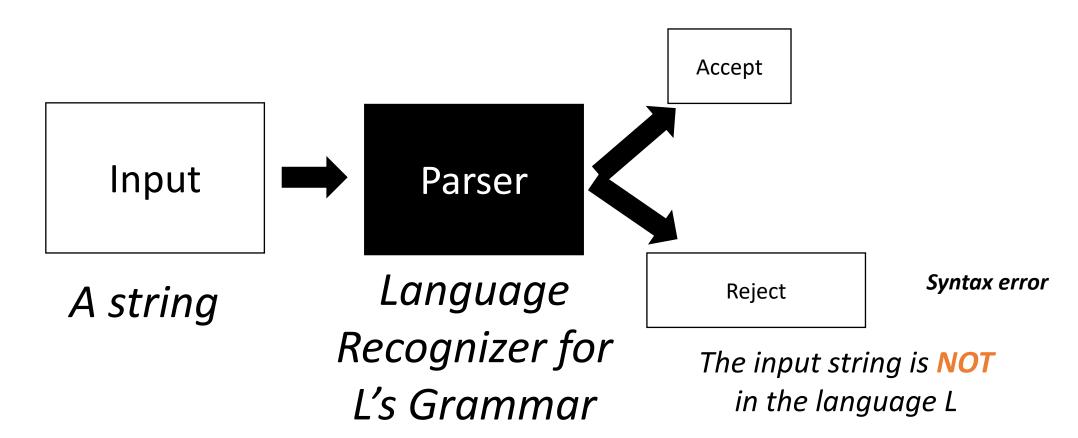




The input string satisfies L's grammar Accept Parser Input Language Reject A string Recognizer for The input string is **NOT** L's Grammar in the language L

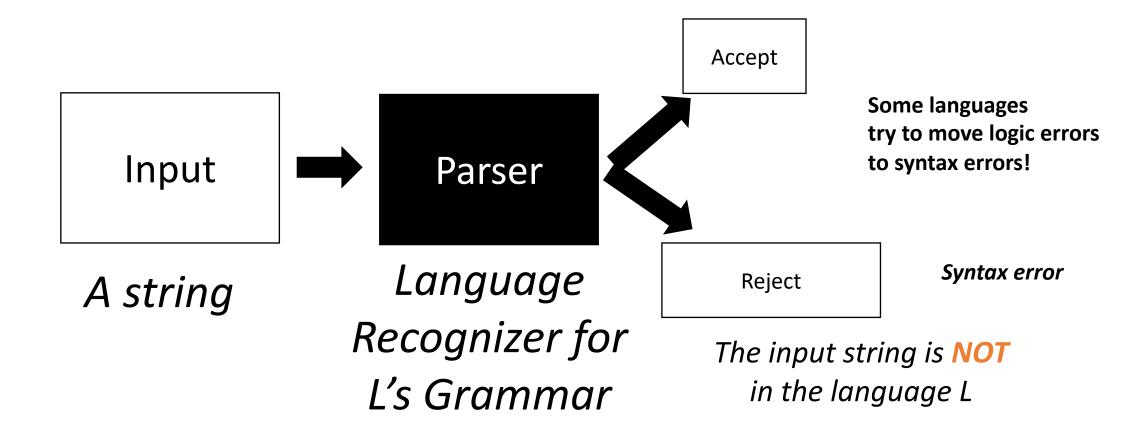
what other types of errors might happen up here?

The input string satisfies L's grammar

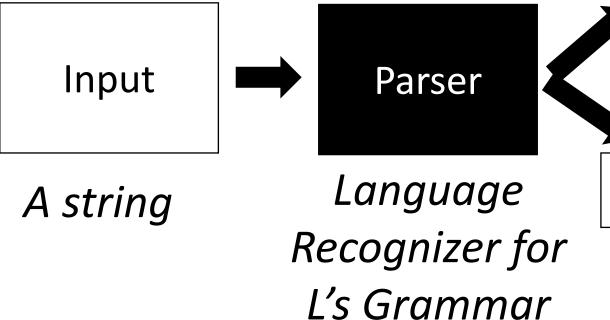


what other types of errors might happen up here?

The input string satisfies L's grammar

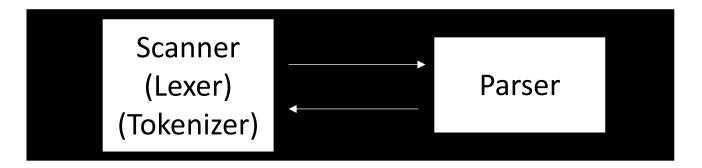


continue to the rest of compilation The input string satisfies L's grammar structured data Accept (e.g. AST) Syntax error Reject The input string is **NOT** in the language L



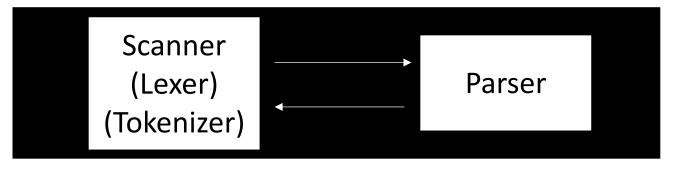
Parser architecture

Parser



Parser architecture

Parser

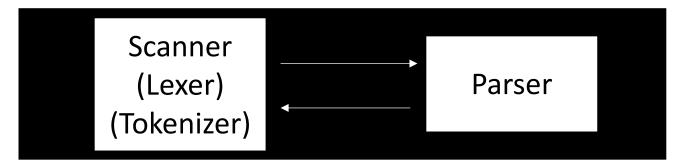


First level of abstraction.
Transforms a string of characters into a string of tokens

Second level: transforms a string of tokens in a tree of tokens.

Parser architecture

Parser



First level of abstraction.
Transforms a string of characters into a string of tokens

Second level: transforms a string of tokens in a tree of tokens.

Language:

Regular Expressions (REs)

Language:

Context-Free Grammars (CFGs)

- List of tokens:
- e.g. {NOUN, ARTICLE, ADJECTIVE, VERB}

My Old Computer Crashed

My Old Computer Crashed



```
[(ARTICLE, "my") (ADJECTIVE, "old") (NOUN, "Computer") (VERB, "Crashed")]
```

My Old Computer Crashed



```
[ (ARTICLE, "my") (ADJECTIVE, "old") (NOUN, "Computer") (VERB, "Crashed")]
```

Lexeme: (TOKEN, value)

• Lets write tokens for arithmetic expression:

$$(5 + 4) * 3$$

ideas?

• Lets write tokens for arithmetic expression:

```
LPAREN = '('
NUMBER = {'5','4','3', ..}

PLUS = '+'

RPAREN = ')'

TIMES = '*'
```

• Lets write tokens for arithmetic expression:

```
LPAREN = '('

NUMBER = {'5','4','3', ...}

PLUS = '+'

RPAREN = ')'

TIMES = '*'

(5 + 4) * 3

LPAREN = '('

NUMBER = {'5','4','3', ...}

OP = {'+', "*"}

RPAREN = ')'
```

• Lets write tokens for arithmetic expression:

```
LPAREN = '('

NUMBER = {'5','4','3', ...}

PLUS = '+'

RPAREN = ')'

TIMES = '*'
```

```
LPAREN = '('
ONE = '1'
TWO = '2'
THREE = '3'
...
PLUS = '+'
RPAREN = ')'
TIMES = '*'
```

You can make tokens more specific

• Lets write tokens for arithmetic expression:

```
LPAREN = '('
NUMBER = {'5','4','3', ..}

PLUS = '+'

RPAREN = ')'

TIMES = '*'
```

```
PAREN = {'(', ')'}

NUMBER = {'5','4','3', ..}

PLUS = '+'

TIMES = '*'
```

- Literal single character:
 - PLUS = '+', TIMES = '*'

- Literal single character:
 - PLUS = '+', TIMES = '*'
- Keyword single string:
 - IF = "if", INT = "int"

- Literal single character:
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 - IF = "if", INT = "int"
- Sets of words:
 - NOUN = {"Cat", "Dog", "Car"}

- Literal single character:
 - PLUS = '+', TIMES = '*'
- Keyword single string:
 - IF = "if", INT = "int"
- Sets of words:
 - NOUN = {"Cat", "Dog", "Car"}
- Numbers
 - NUM = {"0", "1" ...}

- Literal single character:
 - PLUS = '+', TIMES = '*'

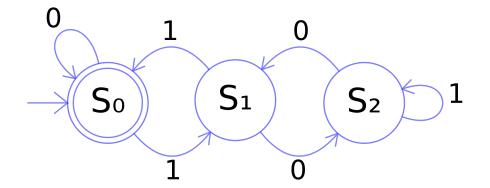
- Keyword single string:
 - IF = "if", INT = "int"

- Sets of words:
 - NOUN = {"Cat", "Dog", "Car"}

Numbers

• Regular expressions!

- Lots of literature!
 - Simplest grammar in the Chomsky language hierarchy
 - abstract machine definition (finite automata)
 - Many implementations (e.g. Python standard library)



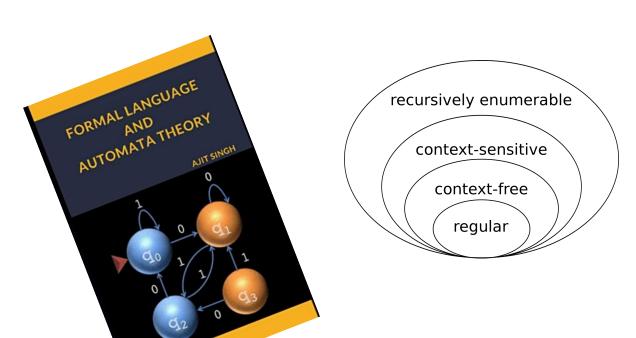


image source: wikipedia

We will define RE's recursively:

Input:

- Regular Expression R
- String *S*

Output:

Does the Regular Expression R match the string S

We will define RE's recursively:

The base case: a character literal

 The RE for a character 'x' is given by 'x'. It matches only the character 'x'

Examples: (demo)

We will define RE's recursively:

Regular expressions are closed under concatenation:

 The concatenation of two REs x and y is given by xy and matches the strings of RE x concatenated with the strings of RE y

Examples (demo)

We will define RE's recursively:

Regular expressions are closed under union:

 The union of two REs x and y is given by x|y and matches the strings of RE x OR the strings of RE y

Examples (demo)

We will define RE's recursively:

Regular expressions are closed under Kleene star:

 The Kleene star of an RE x is given by x* and matches the strings of RE x REPEATED 0 or more times

Examples (demo)

- Use ()'s to force precedence!
- Just like in math:
 - 3 + 4 * 5
- what is the precedence of concatenation, union, and star?
 - "x | yw"
 - Is it "(x | y)w" or "x | (yw)"
 - "xy*"
 - is it (xy)* or x(y*)

- Use ()'s to force precedence!
- Just like in math:
 - 3 + 4 * 5
- what is the precedence of concatenation, union, and star?
 - "x | yw"
 - Is it "(x | y)w" or "x | (yw)"
 - "xy*"
 - is it (xy)* or x(y*)

How can we determine precedence?

• Use ()'s to force precedence!

- Just like in math:
 - 3 + 4 * 5

- what is the precedence of concatenation, union, and star?
 - Star > Concat > Union
 - use () liberally to avoid mistakes!

Most RE implementations provide syntactic sugar:

• Ranges:

- [0-9]: any number between 0 and 9
- [a-z]: any lower case character
- [A-Z]: any upper case character

Optional(?)

- Matches 0 or 1 instances:
- ab?c matches "abc" or "ac"
- can be implemented as: (abc | ac)

Defining tokens using REs

- Literal single character:
 - PLUS = '+', TIMES = '*'
- Keyword single string:
 - IF = "if", INT = "int"
- Sets of words:
 - NOUN = "(Cat) | (Dog) | (Car)"
- Numbers
 - SINGLE_NUM = [0-9]
 - how to do INT = $-?([1-9][0-9]*) \mid 0$
 - how to do FLOAT?

Defining tokens using REs

- Literal single character:
 - PLUS = '+', TIMES = '*'
- Keyword single string:
 - IF = "if", INT = "int"
- Sets of words:
 - NOUN = "(Cat)|(Dog)|(Car)"
- Numbers
 - SINGLE_NUM = [0-9]
 - $INT = -?([1-9][0-9]*) \mid 0$
 - FLOAT =?

Scanner

• Takes in a list of tokens and a string and tokenizes the input

Scanner

Input "My Old Computer Crashed"

Tokens

- ARTICLE = "The | A | My | Your"
- NOUN = "Dog | Car | Computer"
- VERB = "Ran | Crashed | Accelerated"
- ADJECTIVE = "Purple | Spotted | Old"



```
(ARTICLE, "my") (ADJECTIVE, "old") (NOUN, "Computer") (VERB, "Crashed")]
```

Tokens are defined with Regular expressions, which are used to split up the input stream into lexemes

Scanner

Scanner

re.match

- A streaming API supported by most RE libraries
 - Only has to match part the beginning part of the string, not the entire string

re.match

- A streaming API supported by most RE libraries
 - Only has to match part the beginning part of the string, not the entire string
- CLASS_TOKEN = {"cse | 211 | cse211"}
- What would get matched here?: "cse211"
- (CLASS_TOKEN, ?)

Scanners should provide the longest possible match

- Important for operators, e.g. in C
- ++, +=,

how would we parse "x++;"

```
(ID, "x") (ADD, "+") (ADD, "+") (SEMI, ";")

(ID, "x") (INCREMENT, "++") (SEMI, ";")
```

Subtle differences here

- RE definitions are not guaranteed to give you the longest possible match
 - OP = "+ | ++", ID = "[a-z]"
 - What will this return for "x++"
- Scanners will tokenize the string according to the token with the longest match
 - PLUS = "+", PP = "++", ID = "[a-z]"
 - What will this return for "x++"
- What does this mean for you?
 - If you are implementing a scanner?
 - If you are writing tokens?

Scanner Questions?

- Tokens are defined using regular expressions
- A scanner uses tokens to split a string into lexemes
- Regular expressions are good for splitting up a program into numbers, variables, operators, and structure (e.g. parenthesis and braces)
- You will get more practice using them in the homework
- Chapter 2 in EAC goes into detail on regular expression parsing
 - Finite automata etc.

• What about a mathematical sentence (expression)?

limited to non-negative integers and just using + and *

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First lets define tokens:

• What about a mathematical sentence (expression)?

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 - NUM = [0-9]+
 - PLUS = '\+'
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- First lets define tokens:
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What should our language look like?

• What about a mathematical sentence (expression)?

limited to non-negative integers and just using + and *

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'
- What should our language look like?
 - NUM

• What about a mathematical sentence (expression)?

limited to non-negative integers and just using + and *

- First lets define tokens:
 - NUM = [0-9]+
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- What should our language look like?
 - NUM
 - NUM PLUS NUM

limited to non-negative integers and just using + and *

• What about a mathematical sentence (expression)?

- First lets define tokens:
 - NUM = [0-9]+
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 - TIMES = '*'
- What should our language look like?
 - NUM
 - NUM PLUS NUM

• ...

• What about a mathematical sentence (expression)?

limited to non-negative integers and just using + and *

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'
- What should our language look like?
 - NUM
 - NUM PLUS NUM

• ...

Why not just use regular expressions?

What would the expression look like?

Where are we going to run into issues?

What about ()'s

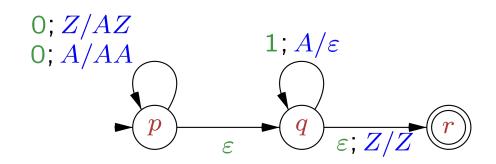
• there is a formal proof available that regex CANNOT match ()'s: pumping lemma

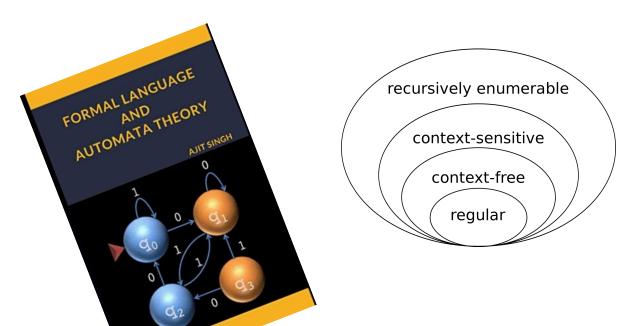
- Informal argument:
 - Try matching $\binom{n}{n}$ using Kleene star
 - Impossible!

 We are going to need a more powerful language description framework!

Context Free Grammars

- Backus–Naur form (BNF)
 - A syntax for representing context free grammars
 - Naturally creates tree-like structures
- More powerful than regular expressions





- cproduction name> : <token list>
 - Example: sentence: ARTICLE NOUN VERB
- cproduction name> : <token list> | <token list>
 - Example:

sentence: ARTICLE ADJECTIVE NOUN VERB | ARTICLE NOUN VERB

Convention: Tokens in all caps, production rules in lower case

Production rules can reference other production rules

non_adjective_sentence: ARTICLE NOUN VERB

adjective_sentence: ARTICLE ADJECTIVE NOUN VERB

sentence: ARTICLE ADJECTIVE* NOUN VERB

sentence: ARTICLE ADJECTIVE* NOUN VERB

We cannot do the star in production rules

- Production rules can be recursive
 - Imagine a list of adjectives:
 "The small brown energetic dog barked"

sentence: ARTICLE adjective_list NOUN VERB

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'

How can we make BNF production rules for this?

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'

expression: NUM

| expression PLUS expression

| expression TIMES expression

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'

Let's add () to the language!

expression: NUM

| expression PLUS expression

expression TIMES expression

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'
 - LPAREN = '\('
 - RPAREN = '\)'

What other syntax like () are used in programming languages?

expression: NUM

| expression PLUS expression

| expression TIMES expression

| LPAREN expression RPAREN

- First lets define tokens:
 - NUM = [0-9]+
 - PLUS = '\+'
 - TIMES = '*'
 - LPAREN = '\('
 - RPAREN = '\)'

What other syntax like () are used in programming languages?

expression: NUM

| expression PLUS expression| expression TIMES expression| LPAREN expression RPAREN

https://stackoverflow.com/questions/1 732348/regex-match-open-tags-exceptxhtml-self-contained-tags

(previously) 2nd most upvoted post on stackoverflow

How to determine if a string matches a CFG?

 A string is accepted by a BNF form if and only if there exists a parse tree.

input: 5

 A string is accepted by a BNF form if and only if there exists a parse tree.

```
expr: NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN
```

 A string is accepted by a BNF form if and only if there exists a parse tree.

```
expr: NUM

| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN
```

 A string is accepted by a BNF form if and only if there exists a parse tree.



 A string is accepted by a BNF form if and only if there exists a parse tree.

input: 5*6

 A string is accepted by a BNF form if and only if there exists a parse tree.

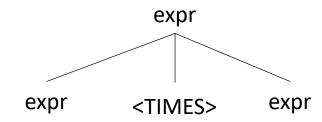
 A string is accepted by a BNF form if and only if there exists a parse tree.

input: 5*6

expr : NUM
| expr PLUS expr

expr TIMES expr

| LPAREN expr RPAREN



 A string is accepted by a BNF form if and only if there exists a parse tree.

expr
expr
</ri>
expr

<NUM, 5>
<NUM, 6>

input: 5*6

 A string is accepted by a BNF form if and only if there exists a parse tree.

input: 5**6

expr: NUM

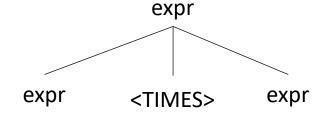
| expr PLUS expr
| expr TIMES expr

| LPAREN expr RPAREN

 A string is accepted by a BNF form if and only if there exists a parse tree.

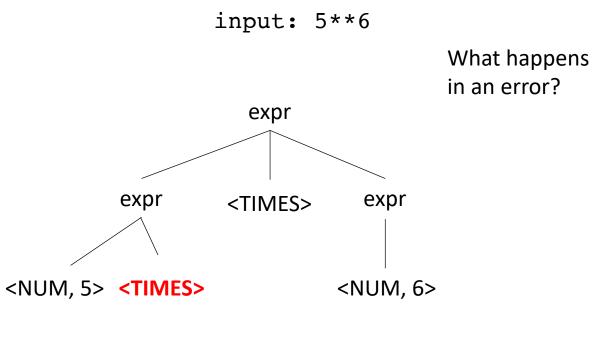
 input: 5**6

What happens in an error?



 A string is accepted by a BNF form if and only if there exists a parse tree.

expr : NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN



Not possible!

 A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: (1+5)*6
```

 A string is accepted by a BNF form if and only if there exists a parse tree.

```
input: (1+5)*6
```

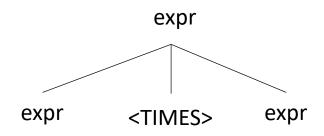
```
expr : NUM expr PLUS expr
```

| expr TIMES expr

| LPAREN expr RPAREN

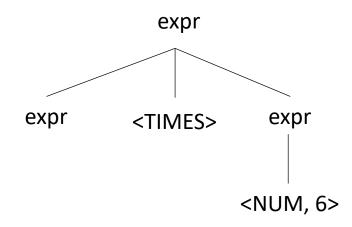
 A string is accepted by a BNF form if and only if there exists a parse tree.

input: (1+5)*6



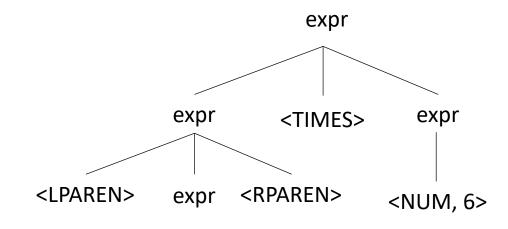
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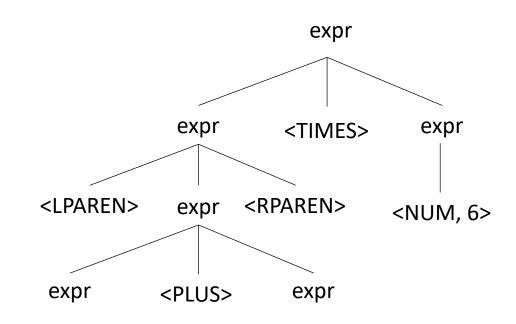
 A string is accepted by a BNF form if and only if there exists a parse tree.

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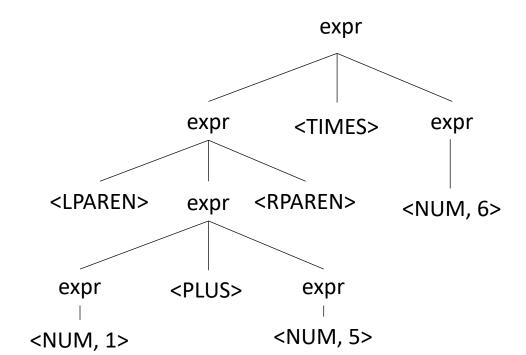
 A string is accepted by a BNF form if and only if there exists a parse tree.

input: (1+5)*6

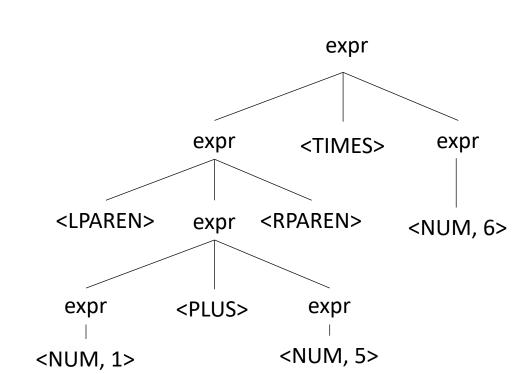


 A string is accepted by a BNF form if and only if there exists a parse tree.

input: (1+5)*6



• Reverse question: given a parse tree: how do you create a string?



input: ?

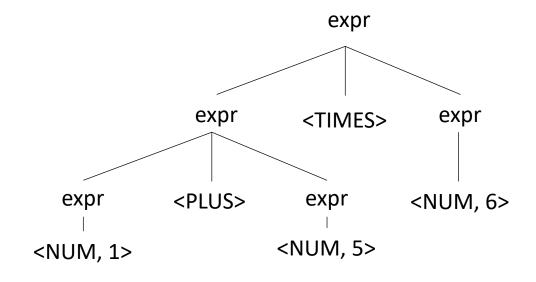
Ambiguous grammars

"I saw a person on a hill with a telescope."

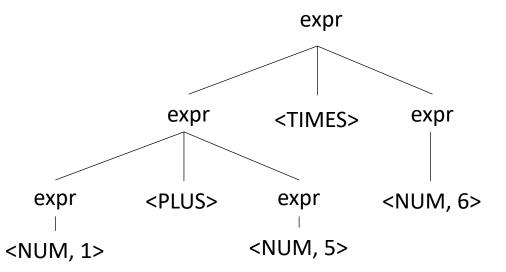
What does it mean??

• Try making a parse tree from: 1 + 5 * 6

• Try making a parse tree from: 1 + 5 * 6



•input: 1 + 5 * 6



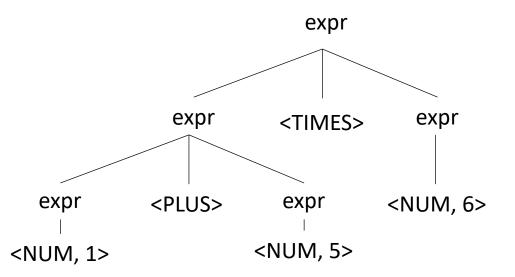
expr: NUM

expr PLUS expr

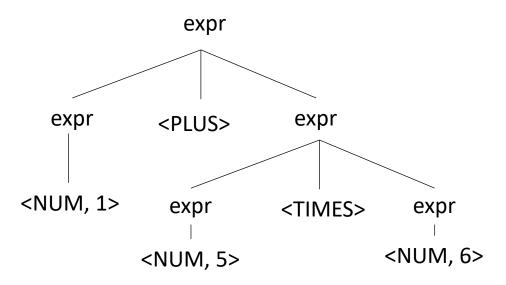
| expr TIMES expr

| LPAREN expr RPAREN

•input: 1 + 5 * 6

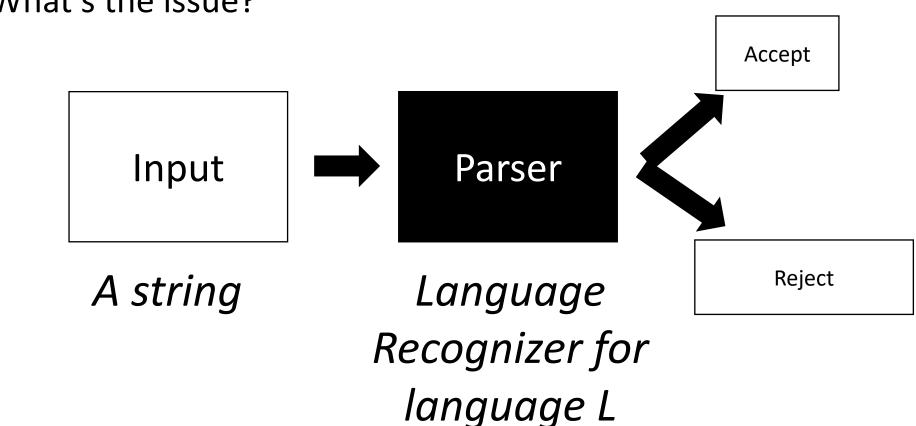


expr : NUM
| expr PLUS expr
| expr TIMES expr
| LPAREN expr RPAREN



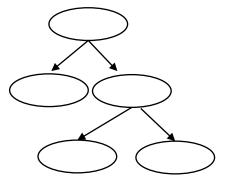
Ambiguous grammars

What's the issue?



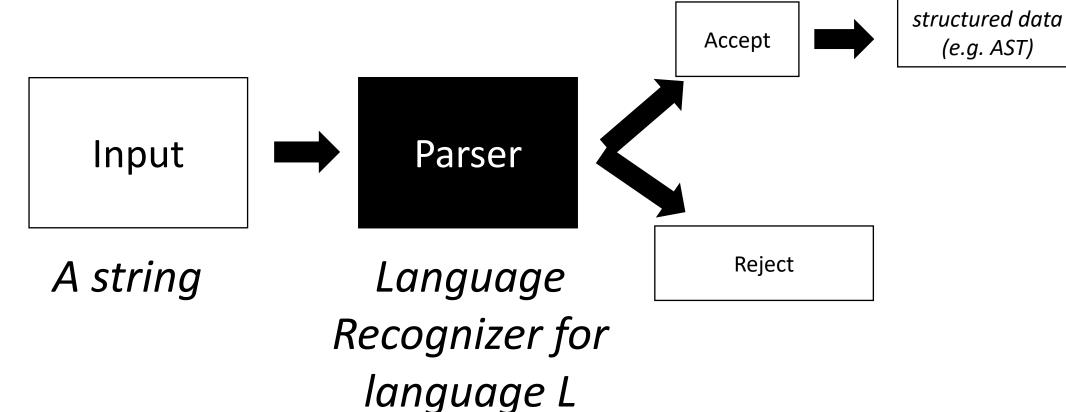
Ambiguous grammars

continue to the rest of compilation



(e.g. AST)

What's the issue?



Meaning into structure

Structural meaning defined to be a post-order traversal

Meaning into structure

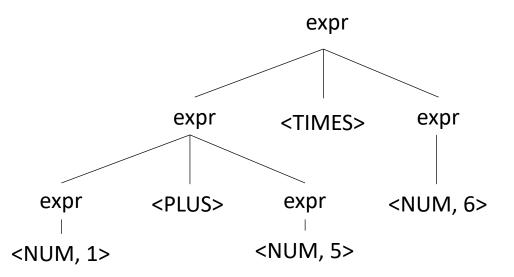
- Structural meaning defined to be a post-order traversal
 - Children return values to their parent
 - Nodes are only evaluated once all their children have been evaluated
 - Evaluated from left to right
 - Also called "Natural Order"

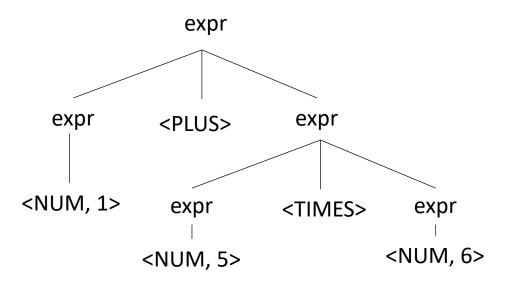
Meaning into structure

- Structural meaning defined to be a post-order traversal
 - Children return values to their parent
 - Nodes are only evaluated once all their children have been evaluated
 - Evaluated from left to right
- Can also encode the order of operation

Ambiguous grammars

•input: 1 + 5 * 6





Avoiding Ambiguity

How to avoid ambiguity related to precedence?

• Define precedence: ambiguity comes from conflicts. Explicitly define how to deal with conflicts, e.g. write* has higher precedence than +

• Some parser generators support this, e.g. Yacc

Avoiding Ambiguity

 How to avoid ambiguity related to precedence?

- Second way: new production rules
 - One rule for each level of precedence
 - lowest precedence at the top
 - highest precedence at the bottom
- Lets try with expressions and the following:
 - + * ()

Avoiding Ambiguity

 How to avoid ambiguity related to precedence?

- Second way: new production rules
 - One rule for each level of precedence
 - lowest precedence at the top
 - highest precedence at the bottom
- Lets try with expressions and the following:

Precedence increases going down

Operator	Name	Productions
+	expr	: expr PLUS expr term
*	term	: term TIMES term factor
()	factor	: LPAREN expr RPAREN NUM

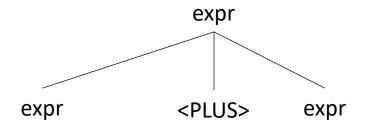
Operator	Name	Productions
+	expr	: expr PLUS expr term
*	term	: term TIMES term factor
()	factor	: LPAREN expr RPAREN NUM

input: 1+5*6

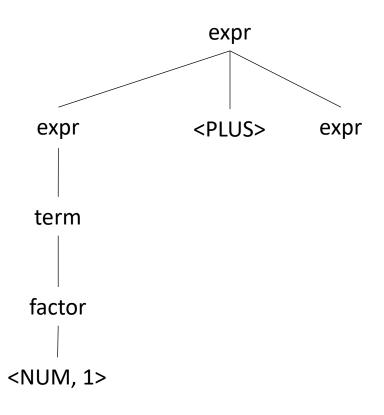
expr

Operator	Name	Productions
+	expr	: expr PLUS expr term
*	term	: term TIMES term factor
()	factor	: LPAREN expr RPAREN NUM

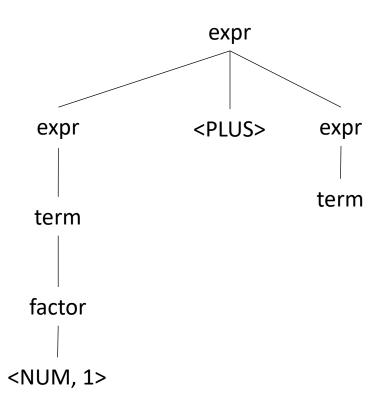
Operator	Name	Productions
+	expr	: expr PLUS expr term
*	term	: term TIMES term factor
()	factor	: LPAREN expr RPAREN NUM



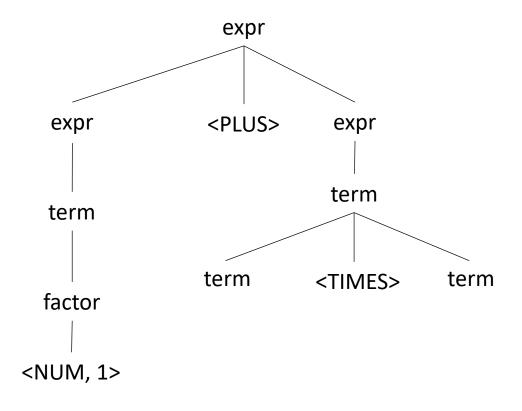
Operator	Name	Productions
+	expr	: expr PLUS expr term
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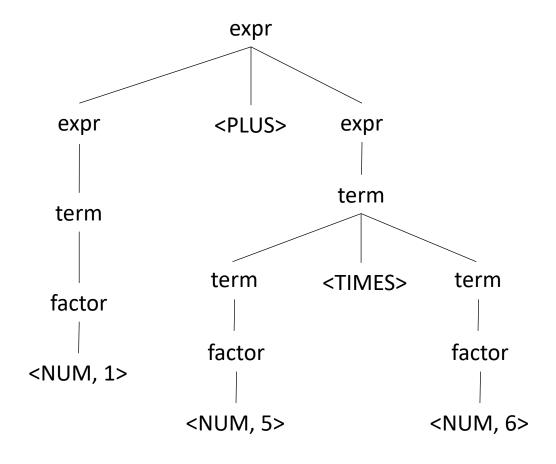
Operator	Name	Productions
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Let's try it for regular expressions, {| . * ()} (where . is concat)

Operator	Name	Productions
1		
•		
*		
()		

Let's try it for regular expressions, {| . * ()} (where . is concat)

Operator	Name	Productions
	union	: union PIPE union concat
	concat	: concat DOT concat starred
*	starred	: starred STAR unit
()	unit	: LPAREN union RPAREN CHAR

Let's try it for regular expressions, {| . * ()}

Operator	Name	Productions
1	union	: union PIPE union concat
	concat	: concat DOT concat starred
*	starred	: starred STAR unit
()	unit	: LPAREN union RPAREN CHAR

input: a.b | c*

Let's try it for regular expressions, {| . * ()}

Operator	Name	Productions
	union	: union PIPE union concat
	concat	: concat DOT concat starred
*	starred	: starred STAR unit
()	unit	: LPAREN union RPAREN CHAR

input: a.b | c*

