CSE211: Compiler Design

Oct. 13, 2021

- **Topic**: Local value numbering

- **Questions**:
  - *What sort of IRs did we talk about last week?*
  - *What were some of the applications of the IRs?*

Announcements

- Homework 1:
  - Due on Monday (at 11:59 pm)
  - Do not count on support from me during the weekends or evenings
  - Office Hours are tomorrow: there will be a sign up sheet

- Updates:
  - Attendance is updated on canvas
  - Docker has all requested SW
  - Let me know if there are issues

Announcements

Next week:

- Wednesday and Friday’s class will be remote:
  - I will be in Chicago
  - I will give a live lecture (zoom link on canvas), I would appreciate it if you attended
  - I will record the lecture and make it available online if you would prefer to attend asynchronously

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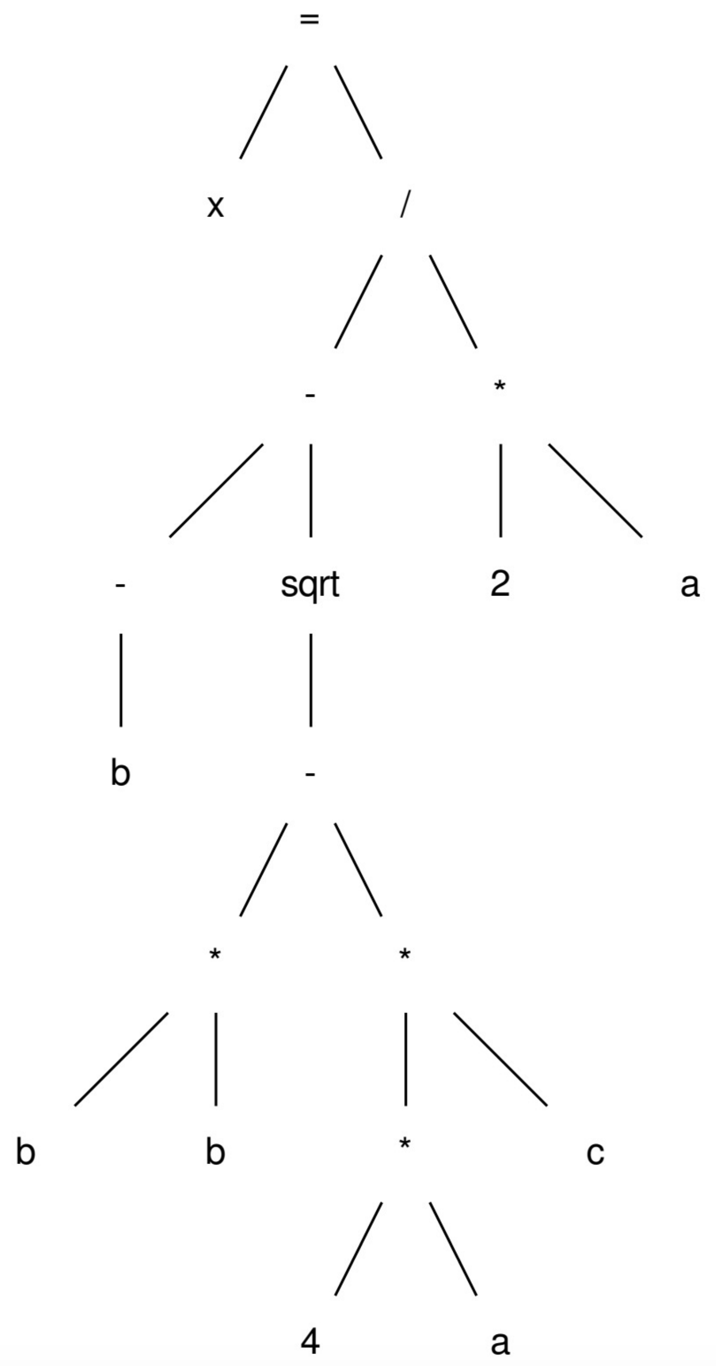
- **Topic**: Local value numbering
- **Questions**:
  - *What sort of IRs did we talk about last week?*
  - *What were some of the applications of the IRs?*

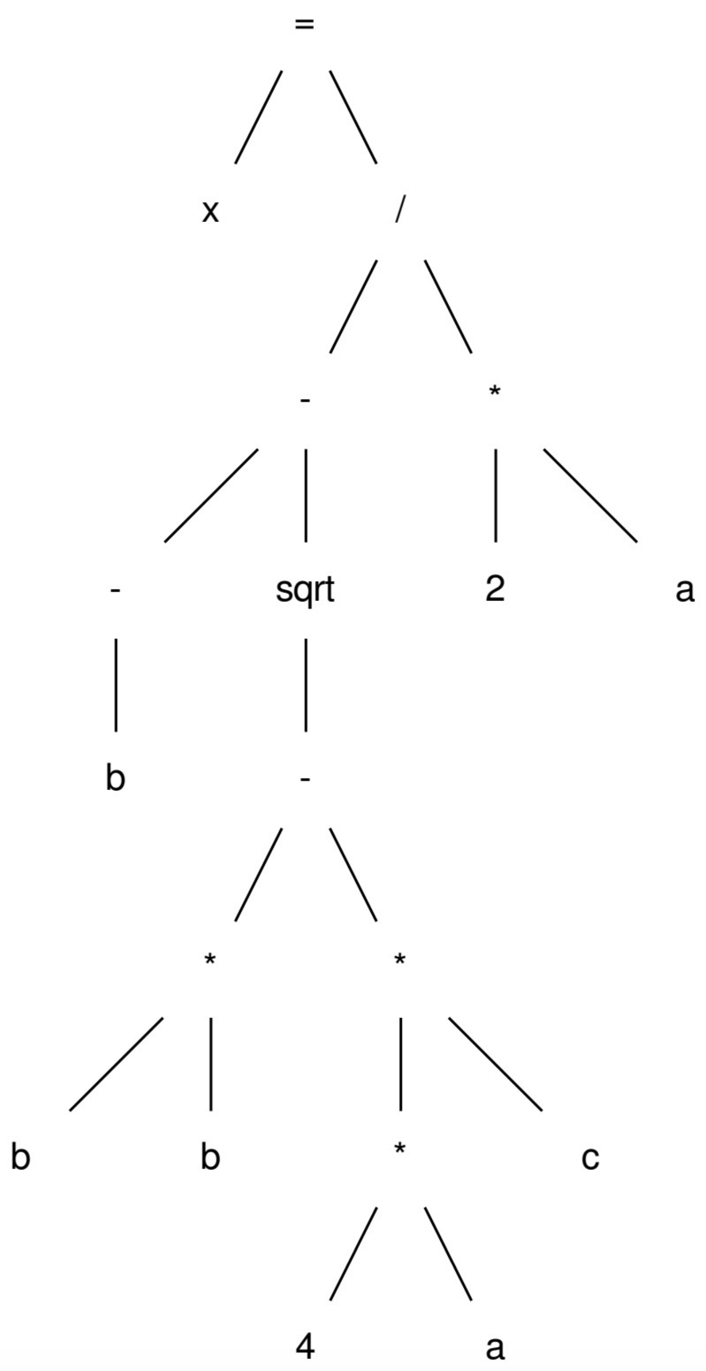
Review IRs:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
x = (-b - sqrt(b*b - 4 * a * c)) / (2*a)
```



```
r0 = neg(b);
r1 = b * b;
r2 = 4 * a;
r3 = r2 * c;
r4 = r1 – r3;
r5 = sqrt(r4);
r6 = r0 – r5;
r7 = 2 * a;
r8 = r6 / r7;
x  = r8;
```



What are some properties of 3 address code?

Control flow in 3 address code

Control flow in 3 address code

Add labels to the 3 address code and have branch instructions

3 address code typically contains a conditional branch:

```
br <reg>, <label0>, <label1>
```

if the value in <reg> is true, branch to <label0>, else branch to <label1>

unconditional branch

```
br <label0>
```

Structure of 3 address code

- What is a basic block?

Structure of 3 address code

- How many basic blocks are in each of the snippets?

```
Label_x:
op1;
op2;
op3;
br label_z;
```

```
Label_x:
op1;
op2;
op3;

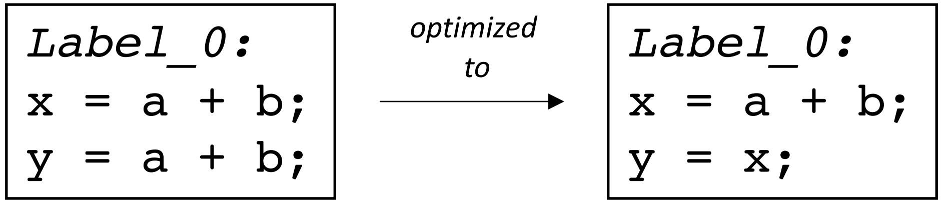
Label_y:
op4;
op5;
```

```
…
if (x) {
   …
}
else {
  …
}
…
```

Local optimizations

- Optimizations that occur in a single basic block
  - What property can we exploit?

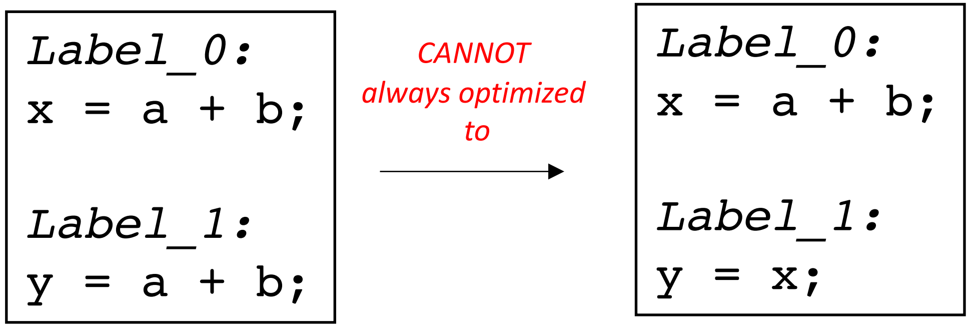
Local optimizations



```
Label_0:
x = a + b;
y = a + b;
```

optimized to

```
Label_0:
x = a + b;
y = x;
```



```
Label_0:
x = a + b;

Label_1:
y = a + b;
```

CANNOT always optimized to

```
Label_0:
x = a + b;

Label_1:
y = x;
```

code could skip Label_0, leaving x undefined!

```
br Label_1;

Label_0:
x = a + b;

Label_1:
y = a + b;
```

Today’s lecture: A local optimization

Local value numbering

- A local optimization over 3 address code
- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
- Can be extended to a regional optimization using flow analysis
  - We will cover in later lectures.

Local value numbering

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```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

Local value numbering

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- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
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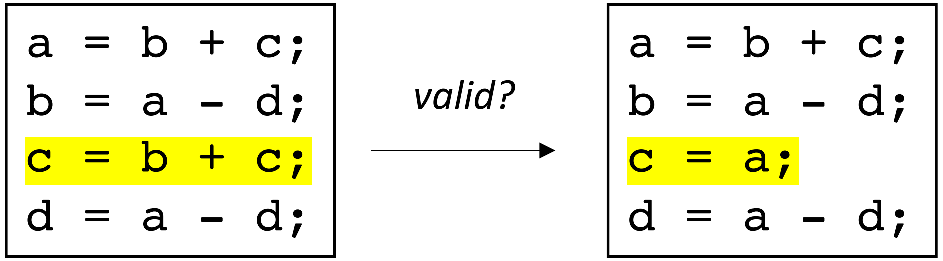

```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

valid?

```
a = b + c;
b = a - d;
c = a;
d = a - d;
```

Local value numbering

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- Can be extended to a regional optimization using flow analysis
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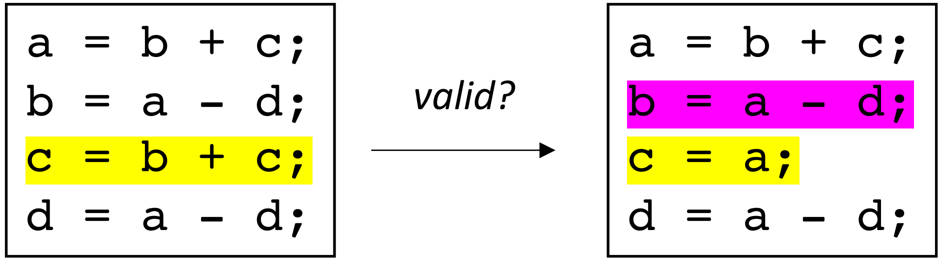


```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

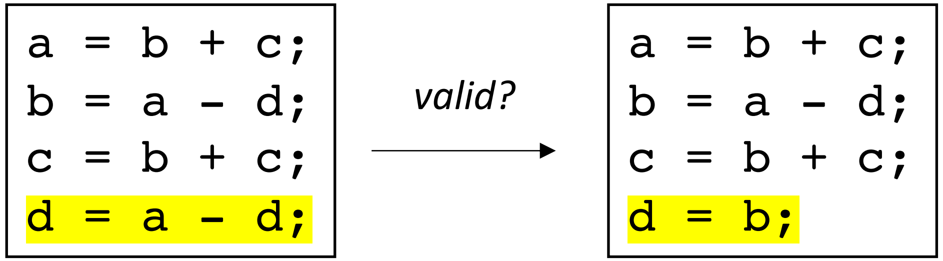
valid?

```
a = b + c;
b = a - d;
c = a;
d = a - d;
```

No! Because b is redefined

Local value numbering

- A local optimization over 3 address code
- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
- Can be extended to a regional optimization using flow analysis
  - We will cover in later lectures.



```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

```
a = b + c;
b = a - d;
c = b + c;
d = b;
```

Local value numbering

- A local optimization over 3 address code

- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)

- Can be extended to a regional optimization using flow analysis
    - We will cover in later lectures.

```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

valid? →

```
a = b + c;
b = a - d;
c = b + c;
d = b;
```

yes!

Local value numbering

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.

- Keep a global counter; increment with new variables or assignments

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

Global_counter = 7

Local value numbering

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.

- Keep a global counter; increment with new variables or assignments

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

Global_counter = 7

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = {
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = {
        “b0 + c1” : “a2”,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
   a2 = b0 + c1;
→  b4 = a2 - d3;
   c5 = b4 + c1;
   d6 = a2 - d3;
```

```
H = {
       “b0 + c1” : a2,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
   a2 = b0 + c1;
→  b4 = a2 - d3;
   c5 = b4 + c1;
   d6 = a2 - d3;
```

```
H = {
        “b0 + c1” : “a2”,
        “a2 - d3” : ”b4”,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
   a2 = b0 + c1;
   b4 = a2 - d3;
→  c5 = b4 + c1;
   d6 = a2 - d3;
```

```
H = {
        “b0 + c1” : “a2”,
        “a2 - d3” : ”b4”,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
  a2 = b0 + c1;
  b4 = a2 - d3;
→ c5 = b4 + c1;
  d6 = a2 - d3;
```

```
H = {
      “b0 + c1” : “a2”,
      “a2 - d3” : ”b4”,
}
```

mismatch due to numberings!

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
   a2 = b0 + c1;
   b4 = a2 - d3;
→  c5 = b4 + c1;
   d6 = a2 - d3;
```

```
H = {
       “b0 + c1” : “a2”,
       “a2 - d3” : ”b4”,
       “b4 + c1” : “c5”,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = {
        “b0 + c1” : “a2”,
        “a2 - d3” : ”b4”,
        “b4 + c1” : “c5”,
}
```

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

- At each step, check to see if the rhs has already been computed.

```
   a2 = b0 + c1;
   b4 = a2 - d3;
   c5 = b4 + c1;
→  d6 = b4;
```

```
H = {
        “b0 + c1” : “a2”,
        “a2 - d3” : ”b4”,
        “b4 + c1” : “c5”,
}
```

match!

What else can we do?

What else can we do?

Consider this snippet:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

Commutative operations

What is the definition of commutative?

Commutative operations

What is the definition of commutative?

```
x OP y == y OP x
```

What operators are commutative? Which ones are not?

Adding commutativity to local value numbering

- For commutative operators (e.g. + *), the analysis should consider a deterministic order of operands.

- You can use variable numbers or lexigraphical order

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

cannot re-order because - is not commutative

```
→ a2 = c1 - b0;
  f4 = d3 * a2;
  c5 = b0 - c1;
  d6 = a2 * d3;
```

```
H = {
       “c1 - b0” : “a2”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
        “c1 - b0” : “a2”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

re-ordered because a2 < d3 lexigraphically

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
        “c1 - b0” : “a2”,
        ”a2 * d3” : “f4”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
    a2 = c1 - b0;
    f4 = d3 * a2;
→   c5 = b0 - c1;
    d6 = a2 * d3;
```

```
H = {
        “c1 - b0” : “a2”,
        ”a2 * d3” : “f4”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
        “c1 - b0” : “a2”,
        ”a2 * d3” : “f4”,
        ”b0 - c1” : “c5”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
   a2 = c1 - b0;
   f4 = d3 * a2;
   c5 = b0 - c1;
→  d6 = a2 * d3;
```

```
H = {
        “c1 - b0” : “a2”,
        ”a2 * d3” : “f4”,
        ”b0 - c1” : “c5”,
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = f4;
```

```
H = {
        “c1 - b0” : “a2”,
        ”a2 * d3” : “f4”,
        ”b0 - c1” : “c5”,
}
```

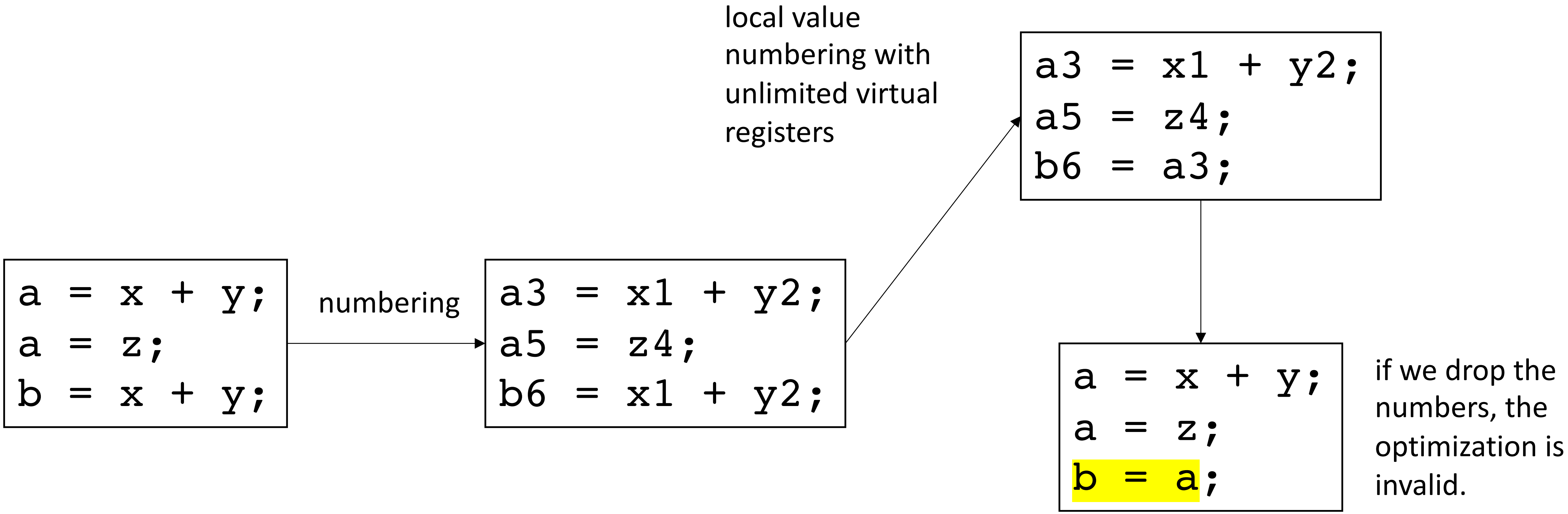
Other considerations?

Local value numbering w/out adding registers

- We’ve assumed we have access to an unlimited number of virtual registers.
- In some cases we may not be able to add virtual registers
  - If an expensive register allocation pass has already occurred.
- New constraint:
  - We need to produce a program such that variables without the numbers is still valid.

Local value numbering w/out adding registers

- Example:



```
a = x + y;
a = z;
b = x + y;
```

numbering

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
```

local value numbering with unlimited virtual registers

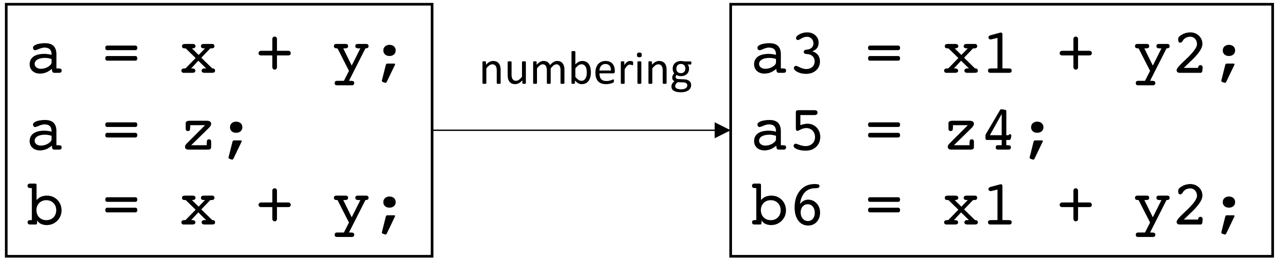
```
a3 = x1 + y2;
a5 = z4;
b6 = a3;
```

```
a = x + y;
a = z;
b = a;
```

if we drop the numbers, the optimization is invalid.

Local value numbering w/out adding registers

- Solutions?



```
a = x + y;
a = z;
b = x + y;
```

numbering

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

We cannot optimize the first line, but we can optimize the second

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

First we number

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
c7 = x1 + y2;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
}
```

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
c7 = x1 + y2;
```

```
H = {
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                   ”a” : 3,
}


H = {
       ”x1 + y2” : ”a3”,
}
```

```
→   a3 = x1 + y2;
    a5 = z4;
    b6 = x1 + y2;
    c7 = x1 + y2;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                  ″a″ : 3,
}
```

```
   a3 = x1 + y2;
→  a5 = z4;
   b6 = x1 + y2;
   c7 = x1 + y2;
```

```
H = {
      ″x1 + y2″ : ″a3″,
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
c7 = x1 + y2;
```

```
Current_val = {
                   ”a” : 5,
}


H = {
       ”x1 + y2” : ”a3”,
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                     ″a″ : 5,
}
```

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
c7 = x1 + y2;
```

```
H = {
      ″x1 + y2″ : ″a3″,
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
  a3 = x1 + y2;
  a5 = z4;
→ b6 = x1 + y2;
  c7 = x1 + y2;
```

```
Current_val = {
                 "a" : 5,
}


H = {
      "x1 + y2" : "a3",
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
   a3 = x1 + y2;
   a5 = z4;
→  b6 = x1 + y2;
   c7 = x1 + y2;
```

```
Current_val = {
                 ″a″ : 5,
                 ″b″ : 6
}

H = {
      ″x1 + y2″ : ″b6″,
}
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                   ″a″ : 5,
                   ″b″ : 6
}

H = {
       ″x1 + y2″ : ″b6″,
}
```

```
  a3 = x1 + y2;
  a5 = z4;
  b6 = x1 + y2;
→ c7 = x1 + y2;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                 ″a″ : 5,
                 ″b″ : 6
}

H = {
      ″x1 + y2″ : ″b6″,
}
```

```
a3 = x1 + y2;
a5 = z4;
b6 = x1 + y2;
c7 = x1 + y2;
```

Local value numbering w/out adding registers

- Keep another hash table to keep the current variable number

```
Current_val = {
                  ″a″ : 5,
                  ″b″ : 6
}

H = {
       ″x1 + y2″ : ″b6″,
}
```

```
  a3 = x1 + y2;
  a5 = z4;
  b6 = x1 + y2;
→ c7 = b6;
```

Anything else we can add to local value numbering?

Anything else we can add to local value numbering?

- Final heuristic: keep sets of possible values

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
Current_val = {
}
```

```
a = x + y;
b = x + y;
a = z;
c = x + y;
```

```
H = {
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
Current_val = {
}
```

```
a3 = x1 + y2;
b4 = x1 + y2;
a6 = z5;
c7 = x1 + y2;
```

```
H = {
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
  a3 = x1 + y2;
  b4 = a3;
  a6 = z5;
→ c7 = x1 + y2;
```

```
Current_val = {
                  "a" : 6,
                  "b" : 4
}
```

```
H = {
   "x1 + y2" : "a3"
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = x1 + y2;
```

```
Current_val = {
                "a" : 6,
                "b" : 4
}
```

```
H = {
    "x1 + y2" : "a3"
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
  a3 = x1 + y2;
  b4 = a3;
  a6 = z5;
→ c7 = x1 + y2;
```

```
Current_val = {
                 “a” : 6,
                 “b” : 4
}
```

```
H = {
   “x1 + y2” : “a3”
}
```

but we could have replaced it with b4!

Local value numbering: value sets

- Final heuristic: keep sets of possible values

rewind to this point

```
a3 = x1 + y2;
b4 = x1 + y2;
a6 = z5;
c7 = x1 + y2;
```

```
Current_val = {
                  “a” : 3,
}
```

```
H = {
   “x1 + y2” : “a3”
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = x1 + y2;
```

```
Current_val = {
                  “a” : 3,
                  ”b” : 4
}


H = {
   “x1 + y2” : [“a3”, “b4”],
}
```

hash a list of possible values

Local value numbering: value sets

- Final heuristic: keep sets of possible values

fast forward again →

```
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = x1 + y2;
```

```
Current_val = {
                  “a” : 6,
                  “b” : 4
}
```

```
H = {
    “x1 + y2” : [“a3”, ”b4”],
}
```

Local value numbering: value sets

- Final heuristic: keep sets of possible values

```
Current_val = {
                  “a” : 6,
                  “b” : 4
}
```

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   “x1 + y2” : [“a3”, ”b4”],
}
```

fast forward
again

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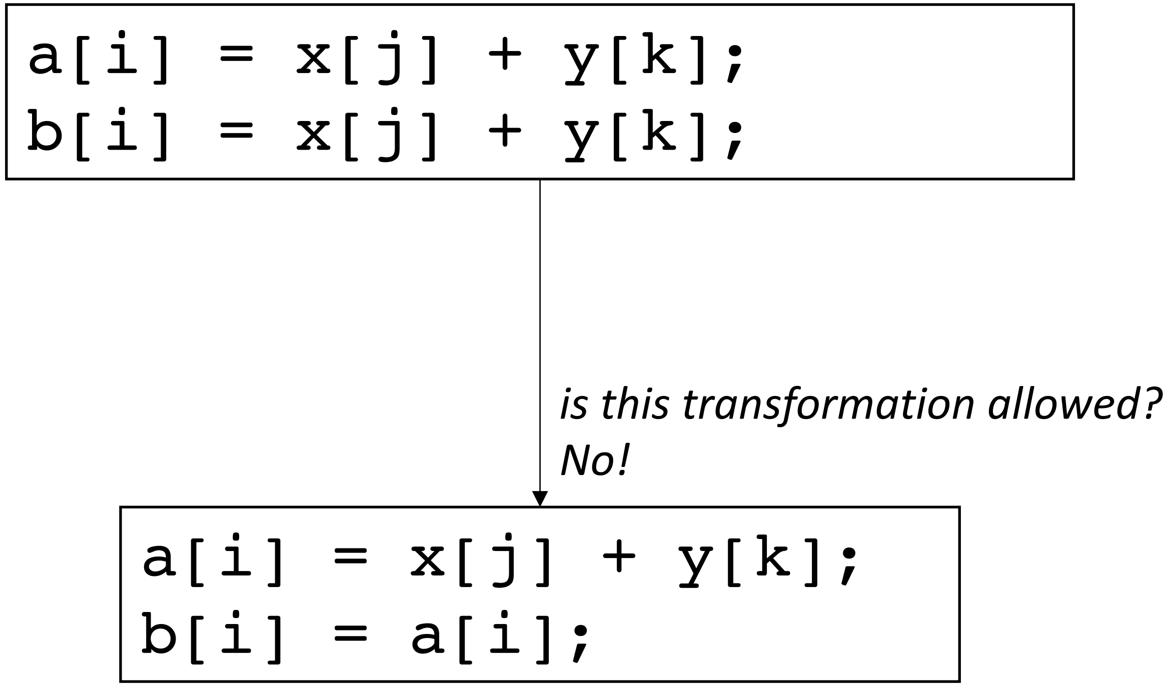
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = b4;
```

Local value numbering: Memory

- Consider a 3 address code that allows memory accesses



```
a[i] = x[j] + y[k];
b[i] = x[j] + y[k];
```

is this transformation allowed?
No!

```
a[i] = x[j] + y[k];
b[i] = a[i];
```

only if the compiler can prove that `a` does not alias `x` and `y`

In the worst case, every time a memory location is updated, the compiler must update the value for all pointers.

Local value numbering: Memory

- How to number:
  - Number each pointer/index pair

```
(a[i],3) = (x[j],1) + (y[k],2);
b[i] = x[j] + y[k];
```

Local value numbering: Memory

- How to number:
  - Number each pointer/index pair

  - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);
(b[i],6) = (x[j],4) + (y[k],5);
```

Local value numbering: Memory

- How to number:
    - Number each pointer/index pair
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```

compiler analysis:

can we trace `a,x,y` to

```
a = malloc(…);
x = malloc(…);
y = malloc(…);
```

// `a,x,y` are never overwritten

Local value numbering: Memory

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programmer annotations can also tell the compiler that no other pointer can access the memory pointed to by a

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`restrict a`

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Local value numbering: Memory

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  - Number each pointer/index pair
  - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);
(b[i],6) = (a[i],3);
```

Optimizing over wider regions

- Local value numbering operated over just one basic block.

- We want optimizations that operate over several basic blocks (a region), or across an entire procedure (global)

- For this, we need Control Flow Graphs and Flow Analysis

On Friday

- Finish up Local value numbering

- Introduce control flow graphs