## CSE110A: Compilers

May 18, 2022

Topics:

- Local value numbering 2
- Loop transformations


## Announcements

- New grades:
- HW 2 posted
- Please let us know within 1 week if there are any issues!
- Pending grades
- Midterm (expect by Monday)
- HW 3 is released
- Due on Tuesday
- Get started if you haven't
- I have office hours tomorrow
- Keep your eye on piazza for this assignment!


## Announcements

- No class on Friday
- Take the time to work on Homework 3!

Quiz

## Quiz

It's the parser's job to perform local value numbering
○ TrueFalse

## Discussion

- Local value numbering operates over 3 address code
- The parser produces 3 address code
- In some cases, the parser might use LVN, but it is independent

$$
\longrightarrow \begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} ; \\
& \mathrm{c} 5=\mathrm{b} 4+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

$$
\begin{aligned}
& \text { н = \{ } \\
& \text { "b0 + c1" : "a2", } \\
& \text { \} }
\end{aligned}
$$

## Quiz

Local value numbering can only work in just one basic block.TrueFalse

## Discussion

- Reminder on a basic block


## Discussion

How might they appear in a high-level language?

How many basic blocks?

- Programs can be split into Basic Blocks:
- A sequence of 3 address instructions such that:
- There is a single entry, single exit

```
|...
```

- Important property: an instruction in a basic block can assume that all preceding instructions will execute

Two Basic Blocks

| Single Basic Block |  |
| :--- | :--- |
| Label_x: <br> op1; <br> op2; <br> op3; <br> br label_z; | op1; <br> op2; <br> op3; <br>  |
| Label_y: <br> op4; <br> op5; |  |

## Discussion

$$
\begin{array}{|l|}
\hline \begin{array}{l}
\text { Label_0: } \\
\mathrm{x}=\mathrm{a}+\mathrm{b} ; \\
\mathrm{y}=\mathrm{a}+\mathrm{b} ;
\end{array} \\
\end{array} \begin{aligned}
& \text { optimized } \\
& \text { to }
\end{aligned} \quad \begin{aligned}
& \text { Label_0: } \\
& \mathrm{x}=\mathrm{a}+\mathrm{b} ; \\
& \mathrm{y}=\mathrm{x} ;
\end{aligned}
$$



$$
\begin{aligned}
& \text { br Label_1; } \\
& \text { Label_0: } \\
& \mathrm{x}=\mathrm{a}+\mathrm{b} ; \\
& \text { Label_1: } \\
& \mathrm{y}=\mathrm{a}+\mathrm{b} ;
\end{aligned}
$$

## Quiz

Local value numbering can only work in just one basic block.TrueFalse

True!
Although you might imagine algorithms that can work on more basic blocks.
This is called superlocal value numbering

Quiz
After perform local value numbering on the following program, how many operations can you save?

$$
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}^{*} f ; \\
& \mathrm{b}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{c}=\mathrm{c}+\mathrm{b} ; \\
& \mathrm{g}=\mathrm{f}^{*} \mathrm{e} ;
\end{aligned}
$$

○ 1
○ 4

Discussion

$$
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e} * \mathrm{f} ; \\
& \mathrm{b}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{c}=\mathrm{c}+\mathrm{b} ; \\
& \mathrm{g}=\mathrm{f} * \mathrm{e} ;
\end{aligned}
$$

$$
\mathrm{H}=\{
$$

$$
\}
$$

## Discussion

$$
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e} * \mathrm{f} ; \\
& \mathrm{b}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{c}=\mathrm{c}+\mathrm{b} ; \\
& \mathrm{g}=\mathrm{f}-\mathrm{e} ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{H}=\{ \\
& \}
\end{aligned}
$$

What if we changed this?

## Quiz

What is a good order of performing the following optimizations (left to right):

1) Local value numbering
2) Loop unrolling
3) Constant propagation

## Discussion

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
    i++;
    x = x + 1;
}
```

how might this influence other optimizations?

## Discussion

```
for (int i = 0; i < 10; i++) {
    x = y + z;
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = y + z;
    i++;
    x = y + z; // can be replaced
        // using LVN
}
```

how might this influence other optimizations?

## Discussion

```
a = 16;
b = a + c;
d = 16;
e = d + c;
```

How might constant propagation change this program

## Discussion

```
a = 16;
b = a + c;
d = 16;
e = d + c;
```

How might constant propagation change this program
$a=16 ;$
$\mathrm{b}=16+\mathrm{c}$;
d $=16$;
$e=16+c$;

## Discussion

```
a = 16;
b = a + c;
d = 16;
e = d + c;
```

How might constant propagation change this program
$\mathrm{a}=16$;
$b=16+c ;$
d $=16$;
$\mathrm{e}=16+\mathrm{c}$; LVN can now replace the bottom one
It's a little more difficult to apply to ClassleR. Do people have any ideas?

## Quiz

What is a good order of performing the following optimizations (left to right):

1) Local value numbering
2) Loop unrolling
3) Constant propagation

Loop unrolling -> Constant propagation -> Local value numbering

## Next quiz

- Teaching feedback from CITL
- Part of a larger program to help improves classes here


## Review

- Basic blocks
- A piece of 3 address code that has one entry and one exit
- Any line of code can assume that all lines before it have been executed
- Allows "local" reasoning
- pycfg example
- Local value numbering
- Local optimization
- Simple algorithm that can be built on:
- initial version just used string comparison
- next we added commutativity
- lastly we extended the algorithm to not add any new registers


## Review

- Algorithm for applying LVN:
- split 3 address code into basic blocks
- for each basic block
- number the variables
- try to remove expensive arithmetic operations


## Today

- Adding constant folding to LVN
- Discussing memory and functions in LVN
- How to add optimized code blocks back into the IR


## Constant propagation and constant folding

- Colloquially, they are often used interchangeably
- Technically (e.g. according to the books)
- Constant propagation is replacing variables with constants
- Constant folding is compile-time evaluation when constants are known


## Constant propagation and constant folding

```
int x = 14;
int y = 7 - x / 2;
return y * (28 / x + 2);
```


## Constant propagation and constant folding

```
int x = 14;
int y = 7-x / 2;
return y * (28 / x + 2);
```

constant propagation

```
int x = 14;
int y = 7-14 / 2;
return y * (28 / 14 + 2);
```


## Constant propagation and constant folding

```
int x = 14;
int y = 7-x / 2;
return y * (28 / x + 2);
```

constant propagation

```
int x = 14;
int y = 7-14 / 2;
return y * (28 / 14 + 2);
```

constant
folding

```
int x = 14;
int y = 0;
return y * (28 / 14 + 2);
```


## Constant propagation and constant folding

```
int x = 14;
int y = 7-x / 2;
return y * (28 / x + 2);
```

```
int \(x=14 ;\)
int \(y=0\);
return 0 * (28 / 14 + 2);
```

constant propagation

```
int x = 14;
int y = 7-14 / 2;
return y * (28 / 14 + 2);
constant
folding
```

```
int x = 14;
int y = 0;
return y * (28 / 14 + 2);
```


## Constant propagation and constant folding

```
int x = 14;
int y = 7-x / 2;
return y * (28 / x + 2);
```

```
int \(x=14 ;\)
int \(y=0\);
return 0 * (28 / 14 + 2);
```

constant propagation

```
int x = 14;
int y = 7-14 / 2;
return y * (28 / 14 + 2);
```


## Typically performed at the same time

```
int x = 14;
int y = 7-x / 2;
return y * (28 / x + 2);
```

constant propagation and folding second line

```
int x = 14;
int y = 0;
return y * (28 / 14 + 2);
```

```
int x = 14;
int y = 0;
return 0;
```

constant propagation and folding third line

## Adding constant folding to LVN

$$
\begin{aligned}
& \mathrm{b}=5 ; \\
& \mathrm{c}=3 ; \\
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{b}=\mathrm{a}-\mathrm{d} ; \\
& \mathrm{c}=\mathrm{a}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{a}-\mathrm{d} ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{H}=\{ \\
& \}
\end{aligned}
$$

$$
\text { Known_values }=\{
$$

## Adding constant folding to LVN

numbering

$$
\begin{aligned}
& \mathrm{b} 0=5 ; \\
& \mathrm{c} 1=3 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 ; \\
& \mathrm{c} 5=\mathrm{a} 2+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

```
H = {
}
Known_values = {
}
```


## Adding constant folding to LVN

As you are iterating through code, add any constant mappings to Known_values:

$$
\begin{aligned}
& \mathrm{b} 0=5 ; \\
& \mathrm{c} 1=3 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 ; \\
& \mathrm{c} 5=\mathrm{a} 2+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{H}=\{ \\
& \} \\
& \text { Known_values = \{ } \\
& \}
\end{aligned}
$$

## Adding constant folding to LVN

As you are iterating through code, add any constant mappings to Known_values:

$$
\begin{aligned}
& \mathrm{b} 0=5 ; \\
& \mathrm{c} 1=3 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 ; \\
& \mathrm{c} 5=\mathrm{a} 2+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{H}=\{ \\
& \}
\end{aligned}
$$

## Adding constant folding to LVN

When you find an arithmetic operation, first check if operands are known

$$
\begin{aligned}
& \mathrm{b} 0=5 ; \\
& \mathrm{c} 1=3 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 ; \\
& \mathrm{c} 5=\mathrm{a} 2+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{H}=\{ \\
& \}
\end{aligned}
$$

## Adding constant folding to LVN

When you find an arithmetic operation, first check if operands are known

$$
\begin{aligned}
& \mathrm{b} 0=5 ; \\
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& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 ; \\
& \mathrm{c} 5=\mathrm{a} 2+\mathrm{c} 1 ; \\
& \mathrm{d} 6=\mathrm{a} 2-\mathrm{d} 3 ;
\end{aligned}
$$

$$
\mathrm{H}=\{
$$

$$
5+3
$$

evaluate and add to known values

```
Known_values = {
"b0" : 5
"c1" : 3
}
```


## Adding constant folding to LVN

When you find an arithmetic operation, first check if operands are known

$$
\begin{aligned}
& \mathrm{b} 0=5 \text {; } \\
& \text { c1 = 3; } \\
& \mathrm{a} 2=8 \text {; } \\
& \mathrm{b} 4=\mathrm{a} 2-\mathrm{d} 3 \text {; } \\
& 5+3 \\
& \mathrm{H}=\{ \\
& \text { evaluate and add to known values } \\
& c 5=a 2+c 1 ; \\
& d 6=a 2-d 3 ;
\end{aligned}
$$

## Adding constant folding to LVN

When you find an arithmetic operation, first check if operands are known

```
b0 = 5;
c1 = 3;
a2 = 8;
b4 = 8 - d3;
c5 = a2 + c1;
d6 = a2 - d3;
```

propagate constant (if IR allows it)

```
H = {
}
Known_values = {
"b0" : 5
"c1" : 3
"a2" : 8

\section*{Adding constant folding to LVN}

When you find an arithmetic operation, first check if operands are known
```

b0 = 5;
c1 = 3;
a2 = 8;
b4 = 8 - d3;
c5 = a2 + c1;
d6 = a2 - d3;

```
add to H
why do we want to store 8 here rather than a2?
```

H = {
"8 - d3" : b4
}
Known_values = {
"b0" : 5
"c1" : 3
"a2" : 8

```

\section*{Arithmetic identities}
\[
\begin{aligned}
& \mathrm{b} 0=0 ; \\
& \mathrm{d} 3=1 ; \\
& \mathrm{f} 7=4 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2 * \mathrm{~d} 3 ; \\
& \mathrm{d} 6=\mathrm{e} 5 * \mathrm{f} 7 ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{H}=\{ \\
& \}
\end{aligned}
\]

\section*{Arithmetic identities}
\[
\begin{aligned}
& \mathrm{b} 0=0 ; \\
& \mathrm{d} 3=1 ; \\
& \mathrm{f} 7=4 ; \\
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2 * \mathrm{~d} 3 ; \\
& \mathrm{d} 6=\mathrm{e} 5 * \mathrm{f} 7 ;
\end{aligned}
\]
```

H = {
}
Known_values = {
"b0":0, "d3":1, "f7":4
}

```

\section*{Arithmetic identities}
what can we do here? add a special rule for + that if any side is 0 , you can just drop the 0 .
\[
\begin{aligned}
\mathrm{b} 0 & =0 ; \\
\mathrm{d} 3 & =1 ; \\
\mathrm{f} 7 & =4 ; \\
\mathrm{a} 2 & =\mathrm{b} 0+\mathrm{c} 1 ; \\
\mathrm{b} 4 & =\mathrm{a} 2 * \mathrm{~d} 3 ; \\
\mathrm{d} 6 & =\mathrm{e} 5 * \mathrm{f} 7 ;
\end{aligned}
\]
```

H = {
}
Known_values = {
"b0":0, "d3":1, "f7":4
}

```

\section*{Arithmetic identities}
what can we do here? add a special rule for + that if any side is 0 , you can just drop the 0 .
\[
\begin{aligned}
& \mathrm{b} 0=0 ; \\
& \mathrm{d} 3=1 ; \\
& \mathrm{f} 7=4 ; \\
& \mathrm{a} 2=\mathrm{c} 1 ; \\
& \mathrm{b} 4=\mathrm{a} 2 * \mathrm{~d} 3 ; \\
& \mathrm{d} 6=\mathrm{e} 5 * \mathrm{f} 7 ;
\end{aligned}
\]
```

H = {
}
Known_values = {
"b0":0, "d3":1, "f7":4
}

```

\section*{Other considerations in LVN}
- Memory and functions

\section*{Local value numbering: Memory}
- Consider a 3 address code that allows memory accesses
```

a[i] = x[j] + y[k];
b[i] = x[j] + y[k];

```
is this transformation allowed?
\(a[i]=x[j]+y[k] ;\)
\(b[i]=a[i] ;\)

\section*{Local value numbering: Memory}
- Consider a 3 address code that allows memory accesses
```

a[i] = x[j] + y[k];
b[i] = x[j] + y[k];

```
is this transformation allowed? No!
\(a[i]=x[j]+y[k] ;\)
\(b[i]=a[i] i\)
only if the compiler can prove that a does not alias x and y

In the worst case, every time a memory location is updated, the compiler must update the value for all pointers.

\section*{Local value numbering: Memory}
- Consider a 3 address code that allows memory accesses
```

a[i] = x[j] + y[k];
b[i] = x[j] + y[k];
a[i] = x[j] + y[k];
b[i] = a[i];

```

Example, initially:
\[
\begin{aligned}
& i=j \\
& a=x \\
& y[k]=1 \\
& x[j]=1
\end{aligned}
\]

What does \(\mathrm{b}[\mathrm{i}]\) equal at the end of each computation?

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
```

a[i] = x[j] + y[k];
b = x[j] + y[k];

```

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(\mathrm{x}[\mathrm{j}], 1)+(\mathrm{y}[\mathrm{k}], 2) ; \\
& (\mathrm{b}, 6)=(\mathrm{x}[\mathrm{j}], ?)+(\mathrm{y}[\mathrm{k}], ?) ;
\end{aligned}
\]

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(\mathrm{x}[\mathrm{j}], 1)+(\mathrm{y}[\mathrm{k}], 2) ; \\
& (\mathrm{b}, 6)=(\mathrm{x}[\mathrm{j}], 4)+(\mathrm{y}[\mathrm{k}], 5) ;
\end{aligned}
\]

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
```

(a[i],3) = (x[j],1) + (y[k],2);
(b,6) = (x[j],4) + (y[k],5);
(c,7) = (x[j],4) + (y[k],5);

```

If there is no memory writes between an assignment to a variable then we can do a replacement

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
```

(a[i],3) = (x[j],1) + (y[k],2);
(b,6) = (x[j],4) + (y[k],5);
(c,7) = (b,6);

```

If there is no memory writes between an assignment to a variable then we can do a replacement

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(x[j], 1)+(y[k], 2) ; \\
& (\mathrm{b}[\mathrm{i}], 6)=(x[j], 4)+(y[k], 5) ;
\end{aligned}
\]

A compiler analysis might try to determine that addresses can't alias
can we trace \(\mathrm{a}, \mathrm{x}, \mathrm{y}\) to a = malloc(...);
\(\mathrm{x}=\operatorname{malloc}(. .\).\() ;\)
\(\mathrm{y}=\operatorname{malloc}(. .\).\() ;\)
// \(a, x, y\) are never overwritten

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(x[j], 1)+(y[k], 2) ; \\
& (\mathrm{b}[\mathrm{i}], 6)=(x[j], 1)+(y[k], 2) ;
\end{aligned}
\]
in this case we do not have to update the number

A compiler analysis might try to determine that addresses can't alias
can we trace \(\mathrm{a}, \mathrm{x}, \mathrm{y}\) to a = malloc(...);
\(\mathrm{x}=\operatorname{malloc}(. .\).\() ;\) y = malloc(...);
// \(\mathrm{a}, \mathrm{x}, \mathrm{y}\) are never overwritten

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(x[j], 1)+(\mathrm{y}[\mathrm{k}], 2) ; \\
& (\mathrm{b}[\mathrm{i}], 6)=(x[j], 4)+(\mathrm{y}[\mathrm{k}], 5) ;
\end{aligned}
\]
programmer annotations can also tell the compiler that no other pointer can access the memory pointed to by a

\section*{Local value numbering: Memory}

\section*{- How to number:}
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
\[
\begin{aligned}
& (\mathrm{a}[\mathrm{i}], 3)=(x[j], 1)+(y[k], 2) ; \\
& (\mathrm{b}[\mathrm{i}], 6)=(x[j], 4)+(y[k], 5) ;
\end{aligned}
\]
in this case we do not have to update the number
restrict a
programmer annotations can also tell the compiler that no other pointer can access the memory pointed to by a

Warning: the compiler does not enforce this!

\section*{Local value numbering: Memory}
- How to number:
- Number each pointer/index pair
- Any pointer/index pair that might alias must be incremented at each instruction
```

(a[i],3) = (x[j],1) + (y[k],2);
(b[i],6) = (a[i],3);

```

Local value numbering: functions

\section*{Local value numbering: functions}

How to number?
\[
\begin{aligned}
& \mathrm{a}=\mathrm{foo}(\mathrm{x}) ; \\
& \mathrm{x}=\mathrm{b} ; \\
& \mathrm{c}=\mathrm{foo}(\mathrm{x}) ;
\end{aligned}
\]

\section*{Local value numbering: functions}

How to number?
\[
\begin{aligned}
& \mathrm{a}=\mathrm{foo}(\mathrm{x}) ; \\
& \mathrm{x}=\mathrm{b} ; \\
& \mathrm{c}=\mathrm{foo}(\mathrm{x}) ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a} 1=\mathrm{foo}(\mathrm{x} 0) ; \\
& \mathrm{x} 3=\mathrm{b} 2 ; \\
& \mathrm{c} 4=\mathrm{foo}(\mathrm{x} 3) ;
\end{aligned}
\]

What if you had first class functions?

\section*{Local value numbering: functions}

How to number?
\[
\begin{aligned}
& \mathrm{a}=\mathrm{foo}(\mathrm{x}) ; \\
& \mathrm{x}=\mathrm{b} ; \\
& \mathrm{c}=\mathrm{foo}(\mathrm{x}) ;
\end{aligned}
\]
the same way
```

a1 = foo(x0);
x3 = b2;
c4 = foo(x3);

```

\section*{Local value numbering: functions}

How to number?
```

a = foo(x);
c = foo(x);

```
```

a1 = foo(x0);
c2 = foo(x0);

```

\section*{Local value numbering: functions}

How to number?
\[
\begin{aligned}
& a=f \circ o(x) ; \\
& c=f o o(x) ;
\end{aligned}
\]
the same way
a1 \(=\) foo(x0);
a1 \(=\) foo(x0);
c2 \(=\) foo (x0);
c2 \(=\) foo (x0);
\(\mathrm{a} 1=\mathrm{foo}(\mathrm{x} 0) ;\)
\(\mathrm{c} 2=\mathrm{foo}(\mathrm{x} 0) ;\)
```

int count = 0;
int foo(int x) {
count += 1;
return 0;
};

```

How about now?

What if foo had this implementation?

\section*{Local value numbering: functions}

How to number?
\[
\begin{aligned}
& \mathrm{a}=\mathrm{foo}(\mathrm{x}) ; \\
& \mathrm{c}=\mathrm{foo}(\mathrm{x}) ;
\end{aligned}
\]
side effects!
the same way
```

a1 = foo(x0);
c2 = foo(x0);

```
```

int count = 0;

```
int count = 0;
int foo(int x) {
int foo(int x) {
    count += 1;
    count += 1;
    return 0;
    return 0;
};
```

};

```

How about now?

What if foo had this implementation?

\section*{Local value numbering: functions}
\[
\begin{aligned}
& a=f o o(x) ; \\
& c=f o o(x) ; \\
& \text { print (count); }
\end{aligned}
\]
```

a1 = foo(x0);
c2 = al;
print(count);

```
are these two programs the same?
```

int count = 0;
int foo(int x) {
count += 1;
return 0;
};

```

\section*{Local value numbering: functions}
- In C/++, functions are assumed to have side effects
- A function that does not have side effects is called "pure"
- You can annotate a function as pure
- __attribute__( (pure))
- warning: compiler does not check this and you can introduce subtle bugs
- Functional languages tend to have a pure-by-default design. Allows more compiler optimizations, but less control to the programmer.

How to stitch optimized code back into the program

\section*{How to stitch optimized code back into the} program
\[
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g}=\mathrm{b}+\mathrm{c} ;
\end{aligned} \mathrm{l}, \begin{aligned}
& \mathrm{l}=\mathrm{bel} 0: \\
& \mathrm{h}=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k}=\mathrm{a}+\mathrm{g} ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}

\author{
split into basic blocks
}
\[
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g}=\mathrm{b}+\mathrm{c} ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g}=\mathrm{b}+\mathrm{c} ;
\end{aligned}
\]
label_0:
\(h=g+a ;\)
\(\mathrm{k}=\mathrm{a}+\mathrm{g}\);
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h}=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k}=\mathrm{a}+\mathrm{g} ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
number
\[
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{l} a \mathrm{ab} l^{2} 0: \\
& \mathrm{h}=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k}=\mathrm{a}+\mathrm{g} ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g}=\mathrm{b}+\mathrm{c} ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{b} 0+\mathrm{c} 1 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h}=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k}=\mathrm{a}+\mathrm{g} ; \\
& \hline
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{a} 1+\mathrm{g} 0 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
move code on slide to make room
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{b} 0+\mathrm{c} 1 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{a} 1+\mathrm{g} 0 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
optimize
\[
\begin{array}{|l|}
\hline \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
\mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
\mathrm{g} 6=\mathrm{b} 0+\mathrm{c} 1 ;
\end{array}
\]
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{a} 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \hline \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{a} 1+\mathrm{g} 0 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
optimize
\[
\begin{array}{|l|}
\hline \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
\mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
\mathrm{g} 6=\mathrm{b} 0+\mathrm{c} 1 ;
\end{array}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{a} 1+\mathrm{g} 0 ;
\end{aligned}
\]
put together?
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{a} 2 ; \\
& \mathrm{label} 0: \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
optimize
put together?
\[
\begin{array}{|l|}
\hline \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
\mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
\mathrm{g} 6=\mathrm{b} 0+\mathrm{c} 1 ;
\end{array}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{a} 1+\mathrm{g} 0 ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{a} 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{a} 2 ; \\
& \mathrm{label} 0: \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the} program
stitch
part 1: assign original variables their latest values
\[
\begin{aligned}
\mathrm{a} 2 & =\mathrm{b} 0+\mathrm{c} 1 ; \\
\mathrm{d} 5 & =\mathrm{e} 3+\mathrm{f} 4 ; \\
\mathrm{g} 6 & =\mathrm{a} 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
make room on slide
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b} 0+\mathrm{c} 1 ; \\
& \mathrm{d} 5=\mathrm{e} 3+\mathrm{f} 4 ; \\
& \mathrm{g} 6=\mathrm{a} 2 ; \\
& \mathrm{g}=\mathrm{g} 6 ; \\
& \mathrm{d}=\mathrm{d} 5 \\
& \mathrm{a}=\mathrm{a} 2 ;
\end{aligned}
\]
label_0:
\(\mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ;\)
\(\mathrm{k} 3=\mathrm{h} 2 ;\)
\(\mathrm{h}=\mathrm{h} 2 ;\)
\(\mathrm{k}=\mathrm{k} 3 ;\)
what else needs to be done?

\section*{How to stitch optimized code back into the program}
stitch part 2: drop numbers from first use of variables
\[
\begin{aligned}
& a 2=b 0+c 1 ; \\
& d 5=e 3+f 4 ; \\
& g 6=a 2 ; \\
& g=\mathrm{g} ; \\
& d=d 5 \\
& d=a 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { a2 }=b+c ; \\
& d 5=e+f ; \\
& g 6=a 2 ; \\
& g=g 6 ; \\
& d=d 5 \\
& a=a 2 ;
\end{aligned}
\]
label_0:
h2 \(=\mathrm{g} 0+\mathrm{a} 1 ;\)
\(\mathrm{k} 3=\mathrm{h} 2 ;\)
\(\mathrm{h}=\mathrm{h} 2 ;\)
\(\mathrm{k}=\mathrm{k} 3 ;\)
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\bar{g}+\mathrm{a} ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}

Now they can be combined
\[
\begin{aligned}
& a 2=b 0+c 1 ; \\
& d 5=e 3+f 4 ; \\
& g 6=a 2 ; \\
& g=g 6 ; \\
& d=d 5 \\
& d=a 2 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \mathrm{h} 2=\mathrm{g} 0+\mathrm{a} 1 ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]
\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d} 5=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g} 6=\mathrm{a} 2 ; \\
& \mathrm{g}=\mathrm{g} 6 ; \\
& \mathrm{d}=\mathrm{d} 5 \\
& \mathrm{a}=\mathrm{a} 2 ; \\
& \mathrm{l} \text { abel_0: } \\
& \mathrm{h} 2=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]
\[
\begin{aligned}
& \text { label_0: } \\
& \text { h2 }=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]

\section*{How to stitch optimized code back into the program}
\begin{tabular}{|c|c|c|}
\hline & new & is it really optimized? \\
\hline original & \[
\begin{aligned}
\mathrm{a} 2 & =\mathrm{b}+\mathrm{c} ; \\
\mathrm{d} 5 & =\mathrm{e}+\mathrm{f} ; \\
\mathrm{g} 6 & =\mathrm{a} 2 ;
\end{aligned}
\] & It looks a lot longer... \\
\hline \(a=b+c ;\) & \(g=96 ;\) & \\
\hline \(d=e+f ;\) & \(d=d 5\) & \\
\hline \(g=b+c ;\) & \(a=a 2 ;\) & \\
\hline & label_0: & \\
\hline label_0: & \(\mathrm{h} 2=\mathrm{g}+\mathrm{a}\); & \\
\hline \[
h=g+a
\] & \(\mathrm{k} 3=\mathrm{h} 2\); & \\
\hline \(\mathrm{k}=\mathrm{a}+\mathrm{g}\); & \(\mathrm{h}=\mathrm{h} 2\); & \\
\hline & \(\mathrm{k}=\mathrm{k} 3\); & \\
\hline
\end{tabular}

\section*{How to stitch optimized code back into the program}
\begin{tabular}{|c|c|}
\hline & new \\
\hline original & \multirow[t]{9}{*}{\[
\begin{aligned}
& \mathrm{a} 2=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d} 5=\mathrm{e}+\mathrm{f} ; \\
& \mathrm{g} 6=\mathrm{a} 2 ; \\
& \mathrm{g}=\mathrm{g} ; \\
& \mathrm{d}=\mathrm{d} ; \\
& \mathrm{a}=\mathrm{a} 2 ; \\
& \mathrm{l} \text { abel_0: } \\
& \mathrm{h} 2=\mathrm{g}+\mathrm{a} ; \\
& \mathrm{k} 3=\mathrm{h} 2 ; \\
& \mathrm{h}=\mathrm{h} 2 ; \\
& \mathrm{k}=\mathrm{k} 3 ;
\end{aligned}
\]} \\
\hline \(a=b+c ;\) & \\
\hline \(d=e+f ;\) & \\
\hline \(g=b+c ;\) & \\
\hline & \\
\hline label_0: & \\
\hline \(h=g+a ;\) & \\
\hline \(\mathrm{k}=\mathrm{a}+\mathrm{g}\) i & \\
\hline & \\
\hline
\end{tabular}

\author{
is it really optimized? \\ Common pattern for code to get larger, but it will contain patterns that are easier optimize away \\ later passes will minimize copies
}

Loop optimizations

\section*{For loops}
- How do they look in different languages
- C/C++
- Python
- Numpy
- How do Python and Numpy look under the hood?
- The more constrained the for loops are, the more assumptions the compiler can make, but less flexibility for the programmer

\section*{For loops}
- The compiler can optimize For loops if they fit a certain pattern
- When developing a compiler optimization, we start with strict constraints and then slowly relax them and make the optimization more general.
- Sometimes it is not worth relaxing the constraints (code gets too complex)
- If you know the constraints, then often you can write code such that the compiler can recognize the pattern and optimize!

\section*{For loops terminology}
- Loop body:
- A series of statements that are executed each loop iteration
- Loop condition:
- the condition that decides whether the loop body is executed
- Iteration variable:
- A variable that is updated exactly once during the loop
- The loop condition depends on the iteration variable

\section*{Loop unrolling}
- Executing multiple instances of the loop body without checking the loop condition.
```

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement

```
unrolled by a factor of 2
```

for (int i = 0; i < 128; i++) {
// body
}

```
```

for (int i = 0; i < 128; i++) {
// body
i++
// body
}

```

\section*{Loop unrolling conditions}
- Under what conditions can we unroll?

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement
```

for (int i = 0; i < 128; i++) {
// body
}

```

\section*{Loop unrolling conditions}
- Under what conditions can we unroll?
```

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement

```
```

for (int i = 0; i < 128; i++) {
// body
}

```

Validate that we actually have an iteration variable
1. Ihs of assignment statement
2. no assignment to variable in body
3. Ihs of loop condition
4. Ihs of assignment_statement

Do these guarantee we will find an iteration variable?
What happens if we don't find one?

\section*{Loop unrolling conditions}
- Under what conditions can we unroll?
```

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement

```
```

for (int i = 0; i < 128; i++) {
// body
}

```

Validate that we actually have an iteration variable
1. Ihs of assignment statement
2. no assignment to variable in body
3. Ihs of loop condition
4. Ihs of assignment_statement

Do these guarantee we will find an iteration variable?
What happens if we don't find one?

\section*{Loop unrolling conditions}
- Under what conditions can we unroll?

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement
```

for (int i = 0; i < 128; i++) {
// body
}

```

Validate properties of iteration variable 1. ?

\section*{Loop unrolling conditions}
- Under what conditions can we unroll?
```

FOR LPAR assignment_statement expr SEMI assignment_statement RPAR statement

```
```

for (int i = 0; i < 128; i++) {
// body
}

```

Validate properties of iteration variable
1. identify an iteration range (start and end)
2. increment by 1

\section*{See everyone on Monday}
- No class on Friday
- See you on Monday
- Topics: Continue Loop unrolling```

